This assignment is due on Thursday, March 9. In this homework and later ones, please note the following:

- **Bold face** lower case letters, *e.g.* $\mathbf{x}$, usually refer to vectors.
- Upper case letters, *e.g.* $A$, usually refer to matrices.
- Lower case letters, *e.g.* $a$, usually refer to scalars.

1. (Demmel 2.11) Let $A$ be symmetric and positive definite. Show that $|a_{ij}| < (a_{ii}a_{jj})^{1/2}$.

2. (Demmel 2.12) Show that if $Y = \begin{bmatrix} I & Z \\ 0 & I \end{bmatrix}$, where $I$ is an $n \times n$ identity matrix, then $\kappa_F(Y) = \|Y\|_F\|Y^{-1}\|_F = 2n + \|Z\|_F^2$.

3. (Demmel 2.13, modified) Suppose that $\|A - B\|$ is “small” and you have a fast algorithm for solving $Ax = b$. Describe an iterative scheme for solving $By = c$. How fast do you expect your algorithm to converge? Hint: Use iterative refinement. Try to derive an error bound of the form $\|r^{(k)}\| \leq \rho^k\|c\|$, where $r^{(k)}$ is the residual as defined in the algorithm for iterative refinement in the notes, and $\rho < 1$ as long as $\|A - B\|$ is sufficiently small.

4. (Demmel 2.17) Suppose that, in MATLAB, you have an $n \times n$ matrix $A$ and an $n \times 1$ matrix (column vector) $b$. What do $A \backslash b$, $b' / A$, and $A / b$ mean in MATLAB? How does $A \backslash b$ differ from $\text{inv}(A) \ast b$?

5. (Demmel 2.20, modified) Given an $n \times n$ nonsingular matrix $A$, how do you efficiently solve the following problems, using Gaussian elimination with partial pivoting?

   (a) Solve the linear system $A^k \mathbf{x} = \mathbf{b}$, where $k$ is a positive integer.

   (b) Compute $\alpha = \mathbf{c}^T A^{-1} \mathbf{b}$.

   (c) Solve the matrix equation $AX = B$, where $B$ is $n \times m$.

   You should (1) describe your algorithms, (2) present them in pseudocode (using a MATLAB-like language; you should not simply write down the algorithm for Gaussian elimination with partial pivoting), and (3) give the approximate number of floating point operations (also known as “flops”). Use the fact that Gaussian elimination with partial pivoting requires approximately $\frac{2}{3}n^3$ flops, and forward and back substitution each require approximately $n^2$ flops.