MAT 415 DIFFERENTIAL EQUATIONS II
HOMEWORK 1

(1) Recall that the circular cylindrical coordinates $0 \leq \rho < \infty$, $0 \leq \varphi \leq 2\pi$, $-\infty < z < \infty$ and the rectangular coordinates are related by the equations

$$x = \rho \cos \varphi, \ y = \rho \sin \varphi, \ z = z.$$

Also recall the del operator and the Laplacian operator

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \tag{1}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2}$$

where $\hat{x}$ denotes the unit vector on the x-axis. Show that in circular cylindrical coordinates, the del operator (1) and the Laplacian operator (2) are given by

$$\nabla = \frac{\partial}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \varphi}\hat{\varphi} + \frac{\partial}{\partial z}\hat{z}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

(2) Recall that the spherical coordinates $0 \leq r < \infty$, $0 \leq \theta \leq \pi$, and $0 \leq \varphi \leq 2\pi$ and the rectangular coordinates are related by the equations

$$x = r \sin\theta \cos\varphi, \ y = r \sin\theta \sin\varphi, \ z = r\cos\theta.$$

Here, θ is the polar angle and φ is the azimuth angle. Show that in spherical coordinates, the del operator (1) and the

Laplacian operator (2) are given by

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\hat{\varphi}$$

$$\nabla^2 = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial \varphi^2}\right].$$