MAT 415 DIFFERENTIAL EQUATIONS II
HOMEWORK 3

Homework 3 concerns Cylindrical Resonant Cavity (see the lecture notes at `http://www.math.usm.edu/lee/matharchives/?p=876`).

(1) Using separation of variables, show that the mode of E_z satisfying the Helmholtz equation

$$\nabla^2 E_z + \alpha^2 E_z = 0$$

is given by

$$(E_z)_{mnk} = \sum_{m,n} J_m(\gamma_{mn}\rho)e^{\pm im\varphi}[a_{mn}\sin kz + b_{mn}\cos kz].$$

Here, $\alpha^2 = \omega^2\epsilon_0\mu_0 = \frac{\omega^2}{c^2}$.

(2) Show that under the boundary conditions: $\frac{\partial E_z}{\partial z}(z=0) = \frac{\partial E_z}{\partial z}(z=l) = 0$ and $E_z(\rho = a) = 0$, the mode in problem (1) can be written as

$$(E_z)_{mnk} = \sum_{m,n} b_{mn} J_m(\frac{\alpha_{mn}}{a}\rho)e^{\pm im\varphi}\cos\frac{p\pi}{l}z.$$

(3) Show that the resonant frequencies are given by

$$\omega_{mnp} = c\sqrt{\frac{\alpha_{mn}^2}{a^2} + \frac{p^2\pi^2}{l^2}}, \quad \begin{cases} m = 0,1,2,\cdots \\ n = 1,2,3,\cdots \\ p = 0,1,2,\cdots . \end{cases}$$