



THE UNIVERSITY OF
SOUTHERN
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Math Department
Student Handbook

Graduate Program

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1 Introduction

This document address comprehensively the masters degree in mathematics offered through the Department of Mathematics, and the doctoral degree offered through the College of Science and Technology (CoST) through the Computational Science Program (COS), and administered by the Department.

2 The MS and Ph.D. Programs in Mathematics

At the masters level the program is structured so as to allow students with wider interests the opportunity to develop a working background in mathematics, while affording students interested in applied mathematics an opportunity to gain a background that would allow them to continue in a doctoral program in applied and computational mathematics. Thus, the revised masters program is not an applied mathematics program but one that provides enough flexibility so that students whose interests are different from applied mathematics have opportunities for following a more diverse curriculum while those interested in applied mathematics have the opportunity to specialize in this area. Indeed, all MS courses are elective course without any restrictions, except as those devised by the students graduate committee.

The doctoral degree administered by the Department of Mathematics and offered through the College of Science and Technology prepares students more intensively for research and profession careers in computational mathematics, with an emphasis in applied and numerical mathematics. This curriculum provides substantially less flexibility than the masters curriculum, in part because by its nature a doctoral degree involves research specialization.

Consistent with Departmental participation in the College COS Program, the doctoral curriculum in mathematics has been structured to service students who are interested in attaining advanced degrees in applied and computational mathematics, and is designed to provide students with a range of courses that emphasize topics in applied analysis and numerical mathematics.

Both degree offerings break new ground, and represents a change from the traditional offerings at the graduate level at USM in that they establish coordinated applied mathematics program within the Department in which the MS and Ph.D. degree offerings can be unified, particularly in terms of having the MS applied mathematics degree serve as a feeder stream for the Ph.D. degree. Despite this emphasis, the masters program maintains a strong independent program, in which students not interested in pursuing applied mathematics can find a significant range of courses in which to study advanced topics in mathematics.

The graduate mathematics program is designed to provide graduate students with an

intensely focused opportunity for study consisting of courses which emphasize developing foundational skills in analysis, complementing these at the doctoral level with courses that provide exposure to advanced topics in numerical methods in computational mathematical science, numerical analysis, and scientific computing.

3 Admissions and Support

3.1 Admission Requirements

Students interested in the mathematics graduate program can apply for admission with a bachelors, masters, or doctoral degree. Students with an appropriate bachelors degree (BS or BA) can only be admitted into the MS program; they cannot apply directly to the Ph.D. program. The successful student will earn an MS degree in mathematics, and then re-apply for the Ph.D. program, if they are interested in further study, or can terminate with the MS degree in mathematics.

Students with appropriate masters and bachelors degrees (e.g., in mathematics, engineering, etc.) may apply to the graduate program. Students with an appropriate masters degree in mathematics or allied fields, e.g., scientific computing, or computational science, or computational engineering, may choose to apply for admission to the mathematics MS program only if their previous degrees are not in this field.

Applicants must meet the requirements specified by the Graduate School. Go to <http://www.usm.edu/graduateschool/>, follow the links to “Graduate Bulletin”, select the current issue, and look under the section “Admission Requirements and Procedures”, “Master’s Degree” or “Doctoral Degree”. Requirements in addition to these are as follows:

- The undergraduate record should indicate that the applicant satisfies the undergraduate requirements for a major in mathematics at Southern Miss.
- Applicants whose native language is not English must score 580 or higher on the TOEFL.

The department chair can recommend conditional admission for an applicant whose credentials strongly meet all requirements (undergraduate record, GRE, and letters of recommendation) except for one.

3.2 Teaching Assistantships

Teaching assistantships are awarded by the chair upon the recommendation of the Graduate Program Committee. All teaching assistants must enroll in the Mathematics Teaching Seminar (MAT 500) during the spring semester of their first year. The usual period of

full-time support on an assistantship is two years, unless extenuating circumstances exist. Students receiving full-time support may not engage in any outside employment. The duties of a first-year teaching assistant include tutoring in the Mathematics Learning Center and/or serving as a teaching assistant for an instructor of a large section. Those first-year students who have previous teaching experience may be assigned to teach one or two sections of Intermediate Algebra (MAT 099). Second year students usually are assigned as instructor of record for introductory mathematics courses each semester.

4 Student Progress and Degree Requirements

Students intent on participating at the MS level are encouraged to write an MS thesis, which entails enrolling in two additional 3-credit research courses. The MS thesis and the associated research work can form the basis for continuing research by those students wishing to continue in their studies to the Ph.D. in Computational Science. Thus both MS and Ph.D. candidates are required to prepare, at a minimum, a research paper during their course of study; for the student pursuing the masters degree as a terminal degree, this research paper qualifies as the thesis, while for the masters student who is formally intent on qualifying as a doctoral candidate, this research may serve as the formal document for the Prospectus presentation. Students must present and defend their research results before their graduate committees.

4.1 Progression through the Program

As shown in Fig. 1, there are several options available for students at all levels for progressing through the masters and doctoral programs.

- 1) Students may enter with a BS, MS or Ph.D. into the programs of study offered by the Department of Mathematics. Clearly, students advancing along a traditional route, i.e., bachelors in mathematics to masters in mathematics, bachelors in mathematics to doctorate in computational mathematics, or masters in mathematics to doctorate in computational mathematics, are accommodated without any special consideration. Special cases include:
 - Students holding a bachelors degree in a field other than mathematics, but one which is appropriate for further masters level work in mathematics work, can be admitted into the MS degree program. It is only in the most exceptional cases that the Department would consider supporting the candidacy of such students for entry into the Ph.D. program, and in any case would require that the student enter as an MS candidate.

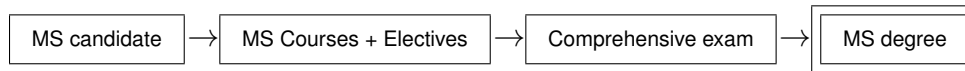
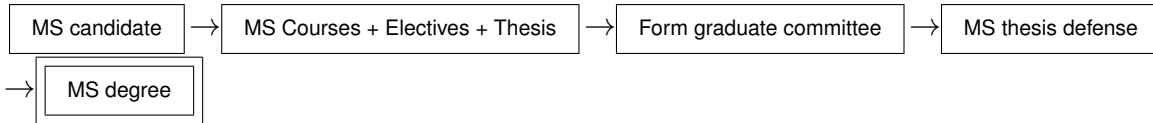
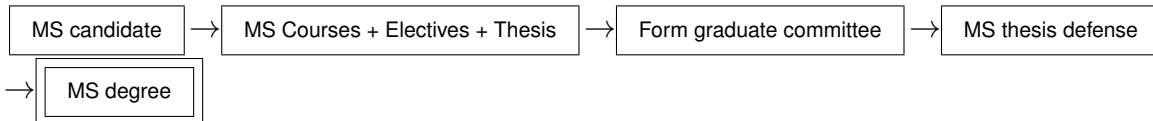
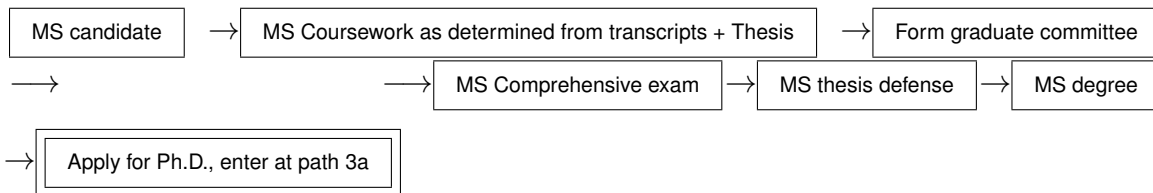
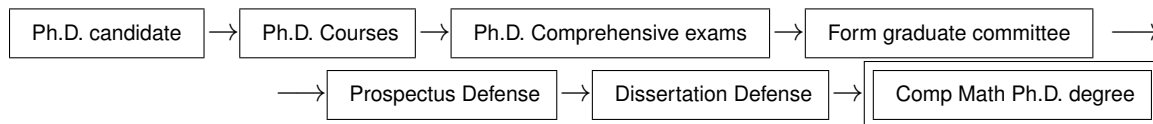
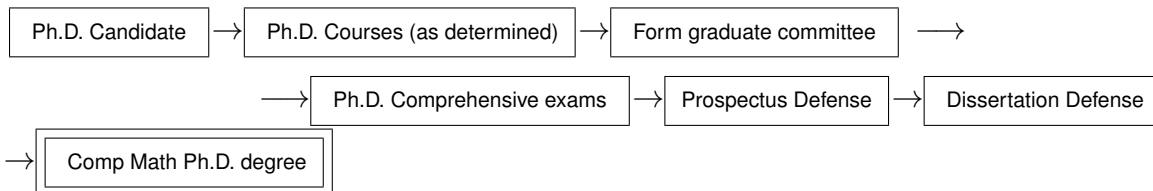
Path 1a: Bachelors applicants for Non-thesis MS option**Path 1b: Bachelors applicants for Thesis MS option****Path 1c: Masters applicants for Thesis MS option. Note incoming student's degree cannot be in mathematics****Path 2: Bachelors applicant for Doctoral Program taking MS degree****Path 3a: Masters applicant for Doctoral Program****Path 3b: Masters applicant for Doctoral Program, transfer student**

Figure 1: *Progress through the Mathematics MS and Computational Mathematics Ph.D. Program. The details discussed are meant to exemplify most options, and is not intended to be exclusive.*

- Students already holding a masters degree in a field other than mathematics, but one which is appropriate for further masters level work in mathematics work, can be admitted for either the MS or Ph.D. degree.
- Students already holding a doctoral degree in a field other than computational mathematics, but one which is appropriate for further doctoral work in computational mathematics, are considered to be Ph.D. transfer students and will have

the curriculum coursework requirements set by their graduate committee.

- 2) Students pursuing an MS by research or Ph.D. degree are encouraged to form their graduate committees as early as possible and the time lines shown for formation of graduate research committees is the latest allowable time. All students must have formed their graduate committees by the end of their first year of study.

4.2 Student graduate committees

The progress of all incoming MS and Ph.D. students is monitored by a faculty advisor. All incoming MS and Ph.D. students will have the same faculty advisor initially, i.e., the Departmental Graduate Director; however, students are strongly encouraged to begin forming a graduate committee within their first semester at the University, and all graduate students are required to have formed their committee by the end of their second semester. A graduate committee typically consists of a chair along with at least two other graduate faculty members. For students following a research degree, i.e., those intent on doing either a masters thesis, or dissertation, the chair of the graduate committee is the student's research advisor.

For research students, the selection of a committee is more critical than for those following a non-research track, thus at the time that a student begins to prepare for their research (either at the MS level when following the MS by research degree) or at the Ph.D. level prior to preparing for the prospectus presentation, the student must form a graduate committee, and the chair of that committee assumes responsibility for monitoring the academic progress of the student, and in particular assumes the responsibility of monitoring and supervising the student's research. Each Ph.D. student must first pass a written Comprehensive exam based on the Ph.D. Core curriculum in order to be allowed to move on to research status.

The research for the Ph.D. constitutes the essence of the doctoral degree. This is first and foremost a degree which requires that the student not only master the state of the art in their chosen area of specialization in computational mathematics, but also extend it. Students who enter the doctoral program, either as masters students or as bachelors students who are nearing completion of their course of study through the MS degree, are encouraged to seek out a faculty research advisor who is willing and able to work with them on a research topic, and who will become the chair of their doctoral committee when this is formed. The sequence of these activities is illustrated in Fig. 1. The timelines are given in the following subsection.

4.3 Degree Requirements

In order to successfully attain the degree MS degree, the student admitted to this course of study must:

1. fulfill the coursework requirements laid down by the Department in regard to the minimum number of credit hours which must be completed and must also satisfy the requirements for satisfactory performance in all of the courses in which they have enrolled;
2. must successfully defend their prospectus for their thesis research by the end of the second semester, if they have chosen the thesis option;
3. pass a written comprehensive exam covering courses in their chosen areas of emphasis as shown in Table 3, if they have chosen the non-thesis option; and
4. successfully present and defend a masters thesis if they have chosen the thesis option.

The comprehensive exam, prospectus defense, thesis defense, and thesis each must be completed by the appropriate deadline posted on the Graduate School web site.

In order to successfully attain the Ph.D. degree, the student admitted to this course of study must:

1. fulfill the coursework requirements laid down by the Department in regard to the minimum number of credit hours which must be completed and must also satisfy the requirements for satisfactory performance in all of the courses in which they have enrolled;
2. must successfully defend their prospectus for their dissertation research by the end of the fourth semester;
3. pass a comprehensive exam covering the core subjects by the end of the sixth semester; and
4. complete and successfully defend their Dissertation which supports their research by the end of the sixth year. The comprehensive exam will cover the core courses associated with the Mathematics Ph.D. degree as shown in Table 2.

The prospectus defense, dissertation defense, and dissertation each must be completed by the appropriate deadline posted on the Graduate School web site.

4.4 University MS and Ph.D. degree requirements

The Department of Mathematics MS curriculum consist of 30 credit hours, which includes an option for graduate research through the writing of a masters thesis. Students who are pursuing the thesis option may take up to two 3 credit course of graduate research (MAT 698) as part of their 30 credit hours. As a rule, courses at or below the 400 level cannot count toward graduate credit, and any course above 500 level can count toward fulfilling the 30-credit hour requirement.

In addition, MS students are required to have completed 6 hours of Advanced Calculus, or equivalent hours, on their undergraduate or graduate transcripts when applying for acceptance into the program. Students who are deficient in this requirement can be granted an acceptance provided they complete the 6 hours of Advanced Calculus (MAT 541 and 542) within the first two semesters of starting their program of study. However, these courses do *not* count toward the 30-credit hour requirement.

Of the 30 credit hours counted toward the MS degree, 21 must be from courses numbered 600 and above, and 18 of those must be mathematics courses. For students who choose the thesis option, the two required instances of MAT 698 count toward this requirement.

While masters students do not have any explicit requirement for participating in Departmental or College seminars or colloquia, including the COS 740 Computational Science seminar series, students, particularly those interested in the MS by research, are strongly encouraged to participate in these opportunities. For students following the thesis option, participation in seminars may be required by a student's graduate committee.

The total hours required by the student to complete the MS degree is 2×3 (Advanced Calculus) $+ 7 \times 3$ (600-level) $+ 3 \times 3$ (Elective) = 36 credit hours. Typically this constitutes a load of 3 graduate courses per semester for 4 semesters. Students supported on teaching assistantships are expected to teach or perform other duties for the Department amounting to 20 hours per week during each Fall and Spring semester.

Students following the course of study leading to a Ph.D. degree are required to take a total of at least 54 credit hours past a masters degree (i.e., a total of 84 credit hours beyond the bachelors degree, of which 30 are in common with the MS curriculum). The doctoral curriculum consists of three 3-credit core courses at the 700 level, 12 credit hours of the dissertation course COS 898, along with three 3-credit hour tools classes, and two 1-credit hour seminar classes, for a total of $3 \times 3 + 12 + 3 \times 3 + 2 = 32$ credit hours. The remaining 22 hours toward the required 54 past the MS degree can be completed through additional graduate-level (500 and above) coursework or additional instances of COS 898.

Foundational MS Courses (24 hrs)	
MAT-641 Functions of a Real Variable I	MAT-642 Functions of a Real Variable II
MAT-636 Functions of a Complex Variable	MAT-601 Differential Geometry
MAT-610 Numerical Linear Algebra	MAT-605 Ordinary Differential Equations
MAT-603 Modern Algebra	MAT-606 Partial Differential Equations

Table 1: MS degree foundational courses. All masters students are required to take 21 hours of 600-level courses, 18 of which must be mathematics courses. These eight courses are generally offered every year or every two years, and are therefore recommended to help fulfill this requirement.

4.5 CoST Ph.D. degree requirements

Three 3-credit hour courses on research methods, or tools classes, is required of all Ph.D. students by the College of Science and Technology. The participating Computational Science departments (e.g., Mathematics or Computer Science) offer these courses, COS 701, COS 702 and COS 703, which are designed to assist the student with technical issues in the area of computing. Topics covered include utilizing computational resources, and using standard tools, software libraries, and techniques for writing scientific publications. For doctoral students with backgrounds deficient in computer science, or in any programming language, it is recommended that elective courses be used to augment skills in programming, and in all cases students entering the doctoral program are expected to have some level of computer skills appropriate for their research interests in computational mathematics as determined by their graduate committee.

The Department also requires all Ph.D. students to register for two semesters, i.e., two 1-credit hours of seminar as offered through COS 740. This course is offered each semester.

4.6 Departmental graduate degree requirements

Progress through the programs for both the MS and Ph.D. follows a similar track. Both MS and Ph.D. candidates are required to pass a comprehensive exam upon completing their respective curriculum. Students who are candidates for the Ph.D. and who successfully pass the comprehensive exams will be admitted to Ph.D. status, and will be required to first successfully present and defend their prospectus for research, and then to defend their dissertation in order to complete the doctorate. Student who are following the MS curriculum are required to successfully present their thesis in order to obtain their degrees. The purpose of the comprehensive exam is to assess mastery of theory by the student. The comprehensive

Core Ph.D. Courses (9 hrs)	Tools Courses (9 hrs)
MAT-771 Functional Analysis MAT-772 Numerical Analysis MAT-773 Signal Analysis	COS-701 Scientific Visualization and Data Mining COS-702 Data Analysis COS-703 Fourier Analysis

Table 2: Ph.D. degree core and tools courses. All doctoral students are required to take these six courses, of which the core courses form the basis for the written comprehensive exam that the student must pass in order to earn a Doctoral degree in Applied and Computational Mathematics. These course constitute a total of 18 credit hours toward the 54 credit hours beyond the MS degree.

exam for both MS courses of study will cover the material in their curriculum. For MS students taking the thesis option, the comprehensive exam will be included in the oral examination during which they defend their thesis. For MS students not taking the thesis option, the comprehensive exam will be a written exam.

The MS thesis option requires two 3 credit research courses or 6 credit hours during which time the student's work will be supervised by a faculty advisor on their project. The MS thesis must be completed before the end of the term in which the student intends to graduate, hence students will be strongly encouraged to develop ideas, and to approach faculty in regard to their research interests early in their studies. As noted, this MS thesis can serve as the basis for the dissertation research conducted by doctoral students, and thus typically serves at the basis for the students prospectus presentation. Ideally, the student should already have begun background reading in research areas which are of interest late in their first semester.

4.7 Academic probation

All grades below a grade of B are unacceptable for graduate work. The following regulations apply to course grade and progress the graduate program:

- All MS and Ph.D. students must have passed MAT 541 Advanced Calculus I and MAT 542 Advanced Calculus II, or equivalent, with a grade of B or higher.
- Only one grade of C, with the exception of MAT 541 and 542, may be counted toward the MS degree.
- No grade of C may be counted toward the Ph.D. degree.

Any graduate student receiving either one grade of F or two grades of C during their program of study will be placed on academic probation. Students placed on probation are permitted to retake any graduate course in which they have earned a grade of C or below. If a grade of B or above is earned in the repeated course, the probationary status will be removed.

If the grade requirements cannot be achieved by a student on academic probation, they are terminated from the program.

4.8 Academic honor code

Students have a responsibility to maintain the academic integrity of the University, and as such students must conduct themselves in a appropriately and in a manner consistent with the University's mission as an institution of higher education. Violations of academic integrity include, but are not limited to: cheating; plagiarism; unapproved multiple submissions; knowingly furnishing false information to any agent of the University for inclusion in academic records; and falsification, forgery, alteration, destruction, or misuse of official University documents, and misuse or abuse of University facilities. Members of the faculty are responsible for announcing the academic requirements of each course, for the conduct of examinations, and for the security of examination papers.

5 The MS Courses

Students are required at a minimum to complete ten 3-credit courses beyond the Advanced Calculus sequence. Seven of these courses must be 600-level or above, of which two may be research classes for students following a thesis option. Students electing a non-thesis option may not substitute thesis or research classes for other coursework.

The courses offered in the masters program are designed to be accessible to students with an undergraduate background that includes, in mathematics: multivariable calculus, linear algebra, and a course in the solution of ordinary differential equations; and, at least one course in using the computer as a technological tool for solving computational problems (e.g., a programming course in C or C++, or using Maple or MATLAB for problem solving). Ideally the student may have had a course in partial differential equations, and a first course in numerical methods would be significant. The student typically will have a bachelors degree in mathematics, scientific computing, physics, or in an applied science such as engineering, or computational engineering.

5.1 Syllabi for Selected Masters Courses

The masters curriculum is focused on introducing the student to some essential ideas in mathematics that cover a large part of modern mathematical thought, and as such is designed to cover fundamental topics in mathematics, with a focus primarily aimed at providing MS students a solid, classical background in analysis and algebra. This is an essential starting point for preparing the student for work and study in applied mathematics, and also provides a useful base for many non-applied fields of mathematics as well.

Each course is 3 credit hours, unless otherwise noted, and each course contains a brief course description which attempts to highlight the major topics which will be covered, showing at the end of each description possible connections with other courses in the curriculum and any prerequisites which may be required. The course descriptions are not meant to be definitive, and are listed in numerical order.

1. **Modern Algebra, MAT 603.** *Foundational concepts associated with the discretization of functions and the elementary numerical techniques used to work with these approximations.*

Topics covered include:

- Elementary concepts from set theory and the integers (introduction);
- Monoids and groups.
- Rings, ideals, and fields. Modules over a principal ideal domain.
- Metric vectors spaces and the classical groups.
- Category theory.
- Group representations.

<i>Curriculum Objectives</i>	This course introduces the student to the essential structures of modern algebra. Learning objectives include proficiency in: 1) working in monoids, groups, rings, fields, modules, and categories; 2) recognizing common terminology of algebra, primarily from group theory: and, 3) describing common and unique properties of different groups and rings.
<i>Research Component</i>	Estimated 20% of class activity, primarily focusing on research articles and topics.
<i>Prerequisite Courses</i>	Modern Algebra II, MAT 424/524.

2. **Differential Equations, MAT 605.** *Extensions of classical theory in the study of ordinary differential equations along with solution techniques.*

Topics covered include:

- Solutions for general homogeneous and inhomogeneous linear systems.
- Real and complex eigenbasis theorems for the constant coefficient linear systems.
- Stability and linear autonomous systems.
- Nonlinear systems and the method of successive approximations.
- The fundamental existence and uniqueness theorem and the maximal interval of existence.
- Stability and nonlinear autonomous systems.
- Periodic orbits and limit cycles in planar autonomous systems.
- Systems depending on a parameter: an introduction of bifurcation.
- Mathematical models in the form of linear and nonlinear systems of equations.

Curriculum Objectives

The theory of differential equations is quite broad. In this, fundamental concepts from the classical theory of ODEs is developed and broadened to include a study of nonlinear systems of ordinary differential equations with focuses on the qualitative behavior and geometrical theory of the solution set. More specifically, the course is to achieve the following goals: 1) To introduce the fundamental theorem and stability theory for linear systems; 2) to develop the fundamental existence-uniqueness theorem for nonlinear systems; 3) to learn about the qualitative behavior and geometrical properties of solutions of linear and nonlinear systems; 4) to introduce systems of differential equations depending on a parameter; and, to study dynamical systems related to the mathematical model of a physical or biological problem.

Research Component

Estimated 10% of class activity, primarily focusing on computer studies.

Prerequisite Courses

Introduction to Differential Equations I, MAT 285.

3. Partial Differential Equations, MAT 606. *The classical theory of partial differential equations along with classical solution techniques.* Topics include:

- Partial differential equations as mathematical models.
- First-order equations and the method of characteristics.
- Classification and canonical form of second-order linear equations in two independent variables.
- The one-dimensional wave equation.

- The method of separation of variables for the wave equation, the heat equation, and elliptic problems.
- Inner product spaces, orthonormal systems, Sturm-Liouville problems and eigenfunction expansions.
- The maximum principle for elliptic equations and the heat equation.
- Green's functions and integral representations.

<i>Curriculum Objectives</i>	To study the partial differential equation problems that arise frequently in engineering and physical applications and to know the foundations of the theory of PDEs for the most fundamental problems in engineering, physics or other sciences. The course is designed to: 1) motivate most mathematical topics by formulating first-order and second-order PDEs from physical principles; 2) classify PDEs into different types and to introduce the theoretical analysis of PDEs for understanding the solution's properties and structure; 3) introduce the integral relations that reduce the number of derivatives required of solutions; and, 4) expose students to the classical techniques of solutions of PDEs including the method of characteristics, eigenfunction expansions, and Green's functions.
<i>Research Component</i>	Estimated 10% of class activity. This course is introductory and covers an extensive range of material.
<i>Prerequisite Courses</i>	Introduction to Differential Equations I, MAT 285.

4. **Numerical Linear Algebra, MAT 610.** *An introduction to foundational concepts in numerical linear algebra emphasizing theory, but with connections to computational applications.* The course covers:

- Common problems in linear algebra. Matrix structure and developing methods for efficiently computing with matrix structures.
- Direct solution methods for linear systems, Gaussian elimination and its variants, including LU decomposition and the Choleksy decomposition. Conditioning, and other numerical difficulties in solving linear systems.
- Gram-Schmidt procedures, orthogonal reduction, QR factorization, discrete Fourier transforms, Fast Fourier Transforms (FFT), and least squares.
- Methods for computing eigenvalues such as the Power method and the QR algorithm. Singular value decomposition. Jacobi methods.

- Iterative solution methods for linear systems including relaxation methods, and the conjugate gradient method. Convergence and rates of convergence of iterative methods.

<i>Curriculum Objectives</i>	This course is intended to cover a broad range of material which includes developing an understanding of the underlying theory motivating numerical linear algebra. Connections to the need to assess computational and algorithmic efficiency form a significant part of the practical applications examined in classwork.
<i>Research Component</i>	Estimated 15% of class activity, primarily focusing on novel algorithmic methods in applied matrix linear algebra.
<i>Prerequisite Courses</i>	MAT 326/426. It is advisable that the student have taken MAT 426/526.

5. **Functions of a Complex Variable I, MAT 636.** *Foundational topics in complex analysis as used in applied mathematics.* Topics covered include:

- Complex numbers.
- Analytic functions and elementary functions.
- Complex integration.
- Series representations for analytic functions.
- Residue theory, and the calculus of residues.
- Conformal mapping.

<i>Curriculum Objectives</i>	The course provides the bridge between the theory of functions of a complex variable and the numerous examples in which this theory is used. The intent is to extend the understanding of analysis using real variables to the complex plane, as well as to introduce some of the unique characteristics of mathematics done in the complex plane, particularly as applied to foundational problems associated with calculus.
<i>Research Component</i>	Estimated 5% of class activity. This course is introductory and covers an extensive range of material.
<i>Prerequisite Courses</i>	Multivariable Calculus, MAT 280.

6. **Functions of a Real Variable I, MAT 641.** *Foundational topics in analysis.*

- Real numbers, measure sets of real numbers, Borel sets of real numbers.
- Measure spaces, Lebesgue measure.

- Metric spaces, Lebesgue-Stieltjes Measures, Hausdorff Measures.
- Measureable functions, sequences of measureable functions, Egoroff's theorem.
- Integration, Fatou's lemma, properties of the integral, and topic of conergence.
- Differentiation.

<i>Curriculum Objectives</i>	This course is intended to provide a rigorous introduction to fundamental ideas in real analysis, primarily focusing on the development of measure theory and building from first principals on a framework of elementary analysis accessible to the beginning graduate student.
<i>Research Component</i>	Estimated 10% of class activity. The focus is on theoretical development, and topics such as the comparison of the development of Rieman and Lebesgue integration provide opportunities for further investigation.
<i>Prerequisite Courses</i>	Advanced Calculus I, MAT 541, and Advanced Calculus II, MAT 542.

7. **Functions of a Real Variable II, MAT 642.** *Foundational topics in analysis.*

- Metric Spaces
- Baire Category Theorem, analytic sets.
- Banach spaces, the L_p spaces, and Hilbert space.

<i>Curriculum Objectives</i>	This course is continues where Functions of a Real Variable I leaves off, developing foundational topics in analysis, leading to the properties of Banach and Hilbert spaces which are central to modern analysis, and which are part of the framework necessary for further study in analysis, or for developing an understanding of topics in numerical and computational analysis. Again, theory is stressed, working from first principals on a framework of elementary analysis accessible to the beginning graduate student.
<i>Research Component</i>	Estimated 10% of class activity. The focus is on theoretical development, and an range of topics can now be examined to ovide opportunities for further investigation.
<i>Prerequisite Courses</i>	Advanced Calculus I, MAT 541, and Advanced Calculus II, MAT 542.

5.2 Specialty Areas for the MS Degree

Courses offered by the department are grouped into nine specialty areas, which are described in Table 3. The student with the help of his or her academic adviser should select a suitable balance in at least three of the nine areas. For the non-thesis student, the master's

<p>Topology/Geometry</p> <p>MAT 572 Modern Geometry MAT 575 General Topology MAT 601 Differential Geometry MAT 683 Topics in Topology/Geometry</p>	<p>Complex Analysis</p> <p>MAT 536 Theory of Functions of a Complex Variable MAT 636 Functions of a Complex Variable MAT 682 Topics in Analysis</p>
<p>Differential Equations</p> <p>MAT 515 Intro to Differential Equations II MAT 517 Intro to Partial Differential Equations MAT 605 Ordinary Differential Equations MAT 606 Partial Differential Equations MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math</p>	<p>Linear Algebra</p> <p>MAT 526 Linear Algebra II MAT 610 Numerical Linear Algebra MAT 681 Topics in Algebra</p>
<p>Computational Mathematics</p> <p>MAT 610 Numerical Linear Algebra MAT 560 Numerical Analysis I MAT 561 Numerical Analysis II MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math</p>	<p>Combinatorics & Graph Theory</p> <p>MAT 537 Graph Theory MAT 539 Combinatorics MAT 629 Applied Combinatorics & Graph Theory</p>
<p>Algebra</p> <p>MAT 521 Number Theory MAT 523 Modern Algebra I MAT 524 Modern Algebra II MAT 603 Modern Algebra MAT 681 Topics in Algebra</p>	<p>Analysis and Probability</p> <p>MAT 520 Probability & Statistics II MAT 641 Functions of a Real Variable I MAT 642 Functions of a Real Variable II MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math</p>
<p>Optimization</p> <p>MAT 518 Linear Programming MAT 519 Optimization in Math Programming MAT 684 Topics in Applied Math MAT 685 Topics in Computational Math</p>	

Table 3: Grouping of master's courses into specialty areas

comprehensive examination will cover the content of two courses (selected by the advisory committee in consultation with the student) from each of the student's three areas of specialization.

5.3 Research classes

A Masters Thesis provides the serious student with an option to demonstrate the ability to work with and implement numerical methods in a computational environment, and to work with abstract concepts in applied mathematics. For more exceptional students, and for those who are interested in pursuing a Ph.D. in Computational Science in the Department of Mathematics, the thesis option provides the opportunity to get started on their research for the doctorate. This course requires that the MS student be taking the thesis option. It can be taken twice for up to 6 credits.

1. **MAT 698, Thesis** *This course requires that the MS student be taking the thesis option. It can be taken twice for up to 6 credits.*
 - Students must arrange to have a graduate faculty member chair their MS thesis committee, and to arrange to have a total of at least three members of the faculty participate on their committees.
 - The thesis due dates are subject to the schedules provided by the Graduate School.

6 Doctoral Courses

Doctoral students are required to complete at least 54 hours past the MS requirement. For the computational mathematics doctoral student, there are three 3-credit hour core courses, and three 3-credit hour tools classes which are required in addition to two 1-credit hour seminar courses run through the College of Science and Technology.

At the doctoral level a substantial amount of time is reserved to the student to pursue an independent course of study as agreed to between the student and their graduate committee. The Department also offers a range of topics course suitable for advanced graduate students in the area of numerical and applied analysis, and these can be supplemented with reading courses on topics of interest to the student and their graduate research advisor. In addition, there are courses offered at the doctoral level by other Departments in the College of Science and Technology. Finally, for students interested in expanding their knowledge of computational topics, there is an extensive curriculum available through the School of Computing, including courses on parallel computing, database structures, and other algorithmic topics that may related to the students work in computational science.

The Ph.D. course outlined should be accessible to students with a strong undergraduate degree in pure or applied mathematics, or with a masters in mathematics. The graduate background should include those topics discussed in Sec 5.1. Students participating in the doctoral program will spend most of their time interacting with their faculty advisor on

research, and so a substantial consideration for any student contemplating the doctorate in Applied and Computational Mathematics is the research interest and activity of the faculty.

While computational science is often regarded as quite applied, it is, at the level of research mathematics, also quite theoretical, often involving fundamental aspects of pure mathematics more than working with the computer. Nevertheless, to fully appreciate that new methods must have computationally utility (or else lead to methods which are), it is important for the student in this field to be able to work with and understand fundamental issues associated with computational science. The capacity to work with a modern computer methods, including compiled languages and the ability to proficiently utilize symbolic tools such as using Maple or MATLAB for problem solving, is important.

The tools courses are important in providing the student with exposure to interdisciplinary issues associated with computational science, and are also designed to provide some exposure, and hence an ability to work with computational system. These courses are not a substitute for gaining needed proficiency in these areas of computational science.

6.1 Syllabi for the Core and Foundational Ph.D. curriculum

The doctoral curriculum provided overlaps with the masters curriculum and extends it, focusing more on topics in applied and computational mathematics. There are only six 3-credit courses required at the doctoral level by the Department of Mathematics. This consist of three required core courses, which are included in the comprehensive exams, and three tools courses, which are not. Of these cores courses, the first continues to develop the underlying theory in applied analysis required to work with and develop advanced numerical methods while the second develops deeper connections computational issues in numerical analysis and scientific computing.

1. **MAT 771, Functional Analysis for Computational Science.** *Functional analysis with applications in applied and computational mathematics*

- Banach spaces and Hilbert spaces.
- Lebesgue integration and the L^p spaces.
- Foundations of linear operator theory.
- Compact sets in Banach spaces.
- The adjoint operator.
- Linear compact operators.
- The spectral theorem.

<i>Curriculum Objectives</i>	This course is an extension of the foundational analytical courses, concepts which are MAT 541/542 Advanced Calculus, I and II, MAT 640 Complex Analysis I, MAT 641 Functions of a Real Variable I, and MAT 642 Functions of a Real Variable II offered to students pursuing a masters in mathematics. This course provides a specialized introduction to functional analysis, selecting topics which are of importance to students pursuing advanced studies in computational and numerical mathematics.
<i>Research Component</i>	Estimated 20% of class activity, dependent on the research interests of the faculty teaching the class.
<i>Prerequisite Courses</i>	Functions of a Real Variable I, MAT 641, and Functions of a Real Variable II, MAT 642.

2. **MAT 772, Numerical Analysis for Computational Sciences** *Advanced methods in numerical analysis for solving discretized systems*

- Error Analysis, fundamental concepts from mathematics and computers.
- Approximation and interpolation redone, with attention to theory as well as computational instabilities.
- Integration techniques.
- Solving nonlinear equations. The theory of fixed point iteration and of relaxation methods, along with some examination of multidimensional issues.
- Numerical solution of ordinary differential equations, with attention to accuracy and stability.
- Iterative methods for large scaled problem, including the conjugate gradient method.

<i>Curriculum Objectives</i>	This course provides a rigorous follow-up to concepts introduced at the undergraduate and graduate levels in MAT 460 Numerical Analysis, in MAT 605 Differential Equations, and MAT 606 Partial Differential Equations, emphasizing the connections between computational mathematics and real and functional analysis. The coursework is designed to put equal emphasis on computational techniques as on theory, and requires the completion of several numerically intensive projects illustrating issues in computational science. Mathematical theory is covered on two levels: 1) foundationally for concepts that are accessible to graduate students, and 2) expository, demonstrating a wealth of techniques and approaches for investigating the behavior of numerical and algorithm issues that arise in computational science.
<i>Research Component</i>	Estimated 25% of class activity.
<i>Prerequisite Courses</i>	MAT 326. It is advisable that the student have taken MAT 426.

3. **MAT 773, Applied Analysis** *Applications of analysis to topics in applied mathematics.*

- The Fourier Transform and distributions.
- The Sobolev spaces and Sobolev inequalities.
- Weak convergence, weak solutions of PDEs.
- Potential theory
- Elliptic regularity, primarily focusing on the solution of Poisson's equation.

<i>Curriculum Objectives</i>	The purpose of this course is to provide connections to advanced topics in applied analysis, particularly as used in the theory of partial differential equations, giving the student exposure to a range of methods and tools for examining issues associated with analytical estimates as well as those associated with convergences.
<i>Research Component</i>	Estimated 10% of class activity, primarily focusing on current literature.
<i>Prerequisite Courses</i>	Complex Analysis, MAT 636, Functions of a Real Variable I, MAT 641, and Functions of a Real Variable II, MAT 642.

6.2 Tools classes for the Ph.D. degree

These courses are offered collectively through the College of Science and Technology for students participating in the Ph.D. Programs in Computational Science. These courses are meant to provide the student with an introduction to the techniques and tools which will be useful in computational science and to provide them the requisite skills to work with numerical concepts. These courses are not meant to displace traditional computer science or mathematics courses. The tools courses consist of:

- COS 701, *Presentation and Visualization Tools*: Working with presentation and computer based tools.
- COS 702, *Data Analysis Tools*: Working with numerical data using basic tools and computational methods which are used to manipulate and analyze large digital data sets. It provides an elementary introduction to using fundamental tools such as signal processing methods for pre- and post-processing data, as well as an elementary introduction to fundamental ideas in numerical and functional approximation to regularize and simplify data.
- COS 703, *Data Storage and Retrieval Tools* Working with file structures in which digital data is stored, and examining the structure of digital data representation including, vector data, and non-numerical data.

Since students participating in the new computational science programs can enter at the MS or Ph.D. levels, these tools courses are constructed so as to be accessible to entry level MS students, and presupposes only a good working knowledge of calculus, the ability to work with basic ideas in algebra and linear algebra, and the ability to work comfortably with computers. There are no computer science programming requirements, however a basic knowledge of computers and experience with using computers and software applications programs is essential.

Ph.D. students are required to enroll in all three 3-credit hour tools courses.

6.3 Seminar Classes

Students enrolled in the Ph.D. program are required to register and attend two 1-credit hour seminars classes, designated as COS 740.