Formulas and Concepts
MAT 099: Intermediate Algebra

Preparing for Tests:
The formulas and concepts here may not be inclusive. You should first take your practice test with no notes or help to see what material you are comfortable with, and what material you may need to review more. Once you have an idea of what material you need to review more, go back to those sections to study.

Preparing for the Final Exam:
Study the individual practice test (Practice Test 1, Practice Test 2, etc.), and spend time on the sections in which you had the lowest scores. After you have prepared thoroughly then you can take the Practice Final Exam. Keep in mind that the Practice Final Exam selects 40 questions at random from your previous practice tests, so it will not necessarily cover all of the material that is on the final.

Things to keep in mind:
Studying for the tests in this course is a process that will require a fairly significant amount of time, and should be completed over the course of a few days. Set aside study time with fellow classmates each day in the days leading up to your exams. If you are unsatisfied with your test scores, you may need to try a different technique for studying. You should contact your instructor to discuss your study habits.

1. Test 1 Material

- **Order of operations**
  Simplify expressions using the order that follows. If grouping symbols such as parentheses are present, simplify expressions with those first, starting with the innermost set. If fraction bars are present, simplify the numerator and the denominator separately.
  1. Evaluate exponential expressions, roots, or absolute values in order from left to right.
  2. Multiply or divide in order from left to right.
  3. Add or subtract in order from left to right.

- **Algebraic Properties**

  Commutative: 
  \[ a + b = b + a \]
  \[ a \cdot b = b \cdot a \]

  Associative:
  \[ (a + b) + c = a + (b + c) \]
  \[ (a \cdot b) \cdot c = a \cdot (b \cdot c) \]

  Distributive: 
  \[ a(b + c) = ab + ac \]

- **Solving linear equations in one variable**
  1. Clear the equation of fractions by multiplying both sides of the equation by the least common multiple
  2. Simplify expressions in parenthesis
  3. Simplify by combining like terms
  4. Move the variable terms to one side and numbers to the other using the addition property of equality
  5. Isolate the variable using the multiplication property (divide both sides by the coefficient of the variable)
  6. Check your answer by substituting into the original equation.

- **Steps for problem solving**
  General Strategy for Problem Solving
  1. **UNDERSTAND** the problem. During this step, become comfortable with the problem. Some ways of doing this are:
     - Read and reread the problem.
     - Propose a solution and check.
     - Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.
     - Construct a drawing
     - **Choose a variable to represent the unknown.** (Very important part)
  2. **TRANSLATE** the problem into an equation.
  3. **SOLVE** the equation.
  4. **INTERPRET** the results; **Check** the proposed solution in the stated problem and **state** your conclusion.

2. Test 2 Material

- A **linear inequality in one variable** is an inequality that can be written in the form \( ax + b < c \), where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \). (The inequality symbols \( \leq, >, \) and \( \geq \) also apply here.)
- **Solving a linear inequality in one variable:**
(1) Clear the equation of fractions.
(2) Remove grouping symbols such as parentheses
(3) Simplify by combining like terms.
(4) Write variable terms on one side and numbers on the other side using the addition property of inequality.
(5) Isolate the variable by dividing both sides by the coefficient of the variable. (Note: if the coefficient is negative, you must also flip the inequality sign)

- **A relation** is a set of ordered pairs.
  (1) The **domain** of the relation is the set of all first components of the ordered pairs.
  (2) The **range** of the relation is the set of all second components of the ordered pairs.
  (3) A **function** is a relation in which each first component in the ordered pairs corresponds to **exactly** one second component.
  (4) **Vertical Line Test**: A relation is a function if no vertical line can be drawn which intersects the function at two distinct points.

- **Function Notation**
  To denote that $y$ is a function of $x$, we can write
  $$ y = f(x) \quad \text{(Read “f of x”)} $$
  This notation means that $y$ is a **function of** $x$ or that $y$ depends on $x$. For this reason, $y$ is called the **dependent variable** and $x$ the **independent variable**.

3. **TEST 3 MATERIAL**

- The slope ($m$) of a line passing through points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the following:
  $$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

- A linear function can be written in the following ways:
  - **Standard Form**: $Ax + By = C$, where $A$, $B$, and $C$ are real numbers.
  - **Slope-Intercept Form**: $y = mx + b$, where $m$ is the slope, and $(0, b)$ is the $y$-intercept.
  - **Point-Slope Form**: $y - y_1 = m(x - x_1)$, where $m$ is the slope, and $(x_1, y_1)$ is some point on the line.

- **Horizontal and Vertical Lines**:
  - $y = c$ **Horizontal Line** The slope is 0, and the $y$-intercept is $(0, c)$
  - $x = c$ **Vertical Line** The slope is undefined, and the $x$-intercept is $(c, 0)$

- The **$x$-intercept** of a line is the point where the graph crosses the $x$ axis, and can be found by letting $y = 0$ (or $f(x) = 0$) and solving for $x$.
- The **$y$-intercept** of a line is the point where the graph crosses the $y$ axis, and can be found by letting $x = 0$.

- **Exponent Rules**: If $a$ and $b$ are real numbers and $m$ and $n$ are integers, then:
  - **Product Rule**: $a^m \cdot a^n = a^{m+n}$
  - **Zero Exponent**: $a^0 = 1 (a \neq 0)$
  - **Quotient Rule**: $\frac{a^m}{a^n} = a^{m-n} (a \neq 0)$
  - **Negative Exponents**: $a^{-n} = \frac{1}{a^n} (a \neq 0)$
  - **Power Rules**: $(a^m)^n = a^{m\cdot n}$
    $$(ab)^m = a^m b^m$$
    $$\left(\frac{a}{b}\right)^m = a^m \frac{1}{b^m}$$

4. **TEST 4 MATERIAL**

- **Polynomials**
  - A polynomial is a finite sum of terms in which all variables have exponents raised to nonnegative integer powers and no variables appear in a denominator.
  - A **term** is a number or the product of a number and one or more variables raised to powers.
  - The **numerical coefficient** of a term is the numerical factor of the term.
- The degree of a term is the sum of the exponents on the variables contained in the term.
- The degree of a polynomial is the largest degree of all its terms.

### General Shapes of Graphs of Polynomial Functions

#### Degree 2
![Coefficient of $x^2$ is a positive number.](image1)

![Coefficient of $x^2$ is a negative number.](image2)

#### Degree 3
![Coefficient of $x^3$ is a positive number](image3)

![Coefficient of $x^3$ is a negative number](image4)

#### Special Products

- **Perfect square trinomial:**
  \[(a + b)^2 = a^2 + 2ab + b^2\]
  \[(a - b)^2 = a^2 - 2ab + b^2\]

- **Difference of two squares:**
  \[(a + b)(a - b) = a^2 - b^2\]

- **Sum and difference of two cubes:**
  \[(a + b)(a^2 - ab + b^2) = a^3 + b^3\]
  \[(a - b)(a^2 + ab + b^2) = a^3 - b^3\]

### Finding the Greatest Common Factor (GCF) of a Polynomial

- **Step 1:** Find the GCF of the numerical coefficients.
- **Step 2:** Find the GCF of the variable factors.
- **Step 3:** The product of the factors found in Steps 1 and 2 is the GCF of the monomials.

### Factor by Grouping

- **Step 1:** Factor out the GCF.
- **Step 2:** Group the terms so that each group has a common factor.
- **Step 3:** Factor out these common factors.
- **Step 4:** Then see if the new groups have a common factor.

### Factor $ax^2 + bx + c$ by trial and check

- **Step 1:** Factor out the GCF.
- **Step 2:** Write all pairs of factors of $ax^2$
- **Step 3:** Write all pairs of factors of $c$, the constant term.
- **Step 4:** Try combinations of these factors until the middle term $bx$ is found.
- **Step 5:** If no combination exists, the polynomial is prime.
5. TEST 5 MATERIAL

- **Difference of two squares:** \( a^2 - b^2 = (a + b)(a - b) \)
- **Perfect square trinomial:**
  \( (a + b)^2 = a^2 + 2ab + b^2 \)
  
  \( (a - b)^2 = a^2 - 2ab + b^2 \)

- **Zero Factor Property**
  If \( a \) and \( b \) are real numbers and \( a \cdot b = 0 \), then \( a = 0 \) or \( b = 0 \). This property is true for three or more factors also.

- **Solve polynomial equations by factoring**
  **Step 1:** Write the equation so that one side is 0.
  **Step 2:** Factor the polynomial completely.
  **Step 3:** Set each factor equal to 0 using the zero factor property.
  **Step 4:** Solve the resulting equations.

- **Pythagorean Theorem**
  In a right triangle, the sum of the squares of the lengths of the two legs \((a \text{ and } b)\) is equal to the square of the length of the hypotenuse.
  \[(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2 \text{ or } a^2 + b^2 = c^2 \]

- **Simplifying or Writing a Rational Expression in Lowest Terms**
  **Step 1:** Completely factor the numerator and denominator of the rational expression.
  **Step 2:** Divide out factors common to the numerator and denominator. (This is the same as “removing a factor of 1”.)

- **Multiplying Rational Expressions** The rule for multiplying rational expressions is
  \[
  \frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS} \text{ as long as } Q \neq 0 \text{ and } S \neq 0
  \]
  To multiply rational expressions, you may use the following steps:
  **Step 1:** Completely factor each numerator and denominator.
  **Step 2:** Use the previous rule and multiply the numerators and denominators.
  **Step 3:** Simplify the product by dividing the numerator and denominator by their common factors.

- **Dividing Rational Expressions** The rule for multiplying rational expressions is
  \[
  \frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR} \text{ as long as } Q \neq 0, S \neq 0, \text{ and } R \neq 0.
  \]
  To divide by rational expressions, use the rule above to change division to multiplication by the reciprocal. Then simplify if possible.

- **Adding or Subtracting Rational Expressions with Common Denominators**
  If \( \frac{P}{Q} \text{ and } \frac{R}{Q} \) are rational expressions, then
  \[
  \frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q} \quad \text{and} \quad \frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}
  \]

- **Finding the Least Common Denominator (LCD)**
  **Step 1:** Factor each denominator completely
  **Step 2:** The LCD is the product of all unique factors each raised to a power equal to the greatest number of times that the factor appears in any factored denominator.

- **Add or subtract rational expressions with unlike denominators**
  **Step 1:** Find the LCD of the rational expressions.
  **Step 2:** Write each rational expression as an equivalent rational expression whose denominator is the LCD found in Step 1.
  **Step 3:** Add or subtract numerators, and write the result over the common denominator.
  **Step 4:** Simplify the resulting rational expression.
6. Test 6 Material

- **Radicals as exponents:** \(a^{1/n} = \sqrt[n]{a}\) if \(\sqrt[n]{a}\) is a real number.

- **Fractions in exponents**
  \[
  a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \\
  a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{(\sqrt[n]{a})^m}
  \]

- **Rules for Radicals:** (Same as for exponents, as we can write radicals as exponents) If \(\sqrt[n]{a}\) and \(\sqrt[n]{b}\) are real numbers
  - **Product Rule:** \(\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}\)
  - **Quotient Rule:** \(\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\left(\frac{a}{b}\right)} (a \neq 0)\)

- **Distance Formula:** The distance \(d\) between points \((x_1, y_1)\) and \((x_2, y_2)\) is the following:
  \[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

- **Midpoint Formula:** The midpoint of the line between points \((x_1, y_1)\) and \((x_2, y_2)\) is the following:
  \[
  \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
  \]