Preparing for Tests:
The formulas and concepts here may not be inclusive. You should first take your practice test with no notes or help to see what material
you are comfortable with, and what material you may need to review more. Once you have an idea of what material you need to review
more, go back to those sections to study.

Preparing for the Final Exam:
Study the individual practice test (Practice Test 1, Practice Test 2, etc.), and spend time on the sections in which you had the lowest
scores. After you have prepared thoroughly then you can take the Practice Final Exam. Keep in mind that the Practice Final Exam
selects 40 questions at random from your previous practice tests, so it will not necessarily cover all of the material that is on the final.

Things to keep in mind:
Studying for the tests in this course is a process that will require a fairly significant amount of time, and should be completed over the
course of a few days. Set aside study time with fellow classmates each day in the days leading up to your exams. If you are unsatisfied
with your test scores, you may need to try a different technique for studying. You should contact your instructor to discuss your study
habits.

• Review
  (1) Pythagorean Theorem: \(a^2 + b^2 = c^2\)
  (2) Motion formula: \(d = rt\)

• Test 1 (1.1-1.5)
  (1) Equation of a circle: \((x - h)^2 + (y - k)^2 = r^2\) where \((h, k)\) is the center, and \(r\) is the radius
  (2) Distance formula: \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
  (3) Midpoint formula: \(\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\)
  (4) Slope: \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
  (5) Point-slope formula: \(y - y_1 = m(x - x_1)\)
  (6) Slope-intercept form: \(y = mx + b\)

Five steps for problem solving:
(a) Familiarize yourself with the problem situation. If the problem is presented in words, this means to read carefully. Some or all of the following can also be helpful.
  (i) Make a drawing, if it makes sense to do so.
  (ii) Make a written list of the known facts and a list of what you wish to find out.
  (iii) Assign variables to represent unknown quantities.
  (iv) Organize the information. Look up a formula, Consult a reference book or an expert in the field, or do research
      on the internet.
  (v) Guess or estimate the answer and check your guess or estimate.
(b) Translate the problem situation to mathematical language or symbolism. For most of the problems you will en-
    counter in algebra, this means to write one or more equations, but sometimes an inequality or some other mathemat-
    ical symbolism may be appropriate.
(c) Carry Out some type of mathematical manipulation. Use your mathematical skills to find a possible solution. In
    algebra, this usually means to solve an equation, an inequality, or a system of equations or inequalities.
(d) Check to see whether your possible solution actually fits the problem situation and is thus really a solution of the
    problem. Although you may have solved an equation, the solution(s) of the equation might not be solution(s) of the
    original problem.
(e) State the answer clearly using a complete sentence.

• Test 2 (1.6, 2.2-2.5)
  (1) Function composition: \((f \circ g)(x) = f(g(x))\)
  (2) Difference quotient: \(\frac{f(x+h) - f(x)}{h}\)
  (3) Symmetry:

Algebraic Tests of Symmetry:
(a) x-axis: If replacing \(y\) with \(-y\) produces an equivalent equation, then the graph is symmetric with respect to the x-axis.
(b) y-axis: If replacing \(x\) with \(-x\) produces an equivalent equation, then the graph is symmetric with respect to the y-axis.
(c) Origin: If replacing \(x\) with \(-x\) and \(y\) with \(-y\) produces an equivalent equation, then the graph is symmetric with
    respect to the origin.

Summary of Transformations of \(y = f(x):\)
(a) **Vertical Translation:** $y = f(x) \pm b$
For $b > 0$:
the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted up $b$ units;
the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted down $b$ units.

(b) **Horizontal Translations:** $y = f(x \pm d)$
For $d > 0$:
the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted right $d$ units;
the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted left $d$ units.

(c) **Reflections:**
Across the $x$-axis:
the graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the $x$-axis.
Across the $y$-axis:
the graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ across the $y$-axis.

(d) **Vertical Stretching and Shrinking:** $y = af(x)$
the graph of $y = af(x)$ can be obtained from the graph of $y = f(x)$ by
stretching vertically for $|a| > 1$, or
shrinking vertically for $0 < |a| < 1$.
For $a < 0$, the graph is also reflected across the $x$-axis.

(e) **Horizontal Stretching and Shrinking:** $y = f(cx)$
the graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by
shrinking horizontally for $|c| > 1$, or
stretching horizontally for $0 < |c| < 1$.
For $c < 0$, the graph is also reflected across the $y$-axis.

- **Test 3 (3.2-3.5)**

1. Standard form of the quadratic equation:
   
   
   $$ax^2 + bx + c = 0$$

2. Quadratic formula:
   
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. Properties of quadratic functions and graphing quadratic functions:
   
   
   ![Graph of a quadratic function](image)

   The graph of the function $f(x) = a(x - h)^2 + k$ is a parabola that
   - opens up if $a > 0$ and down if $a < 0$;
   - has $(h, k)$ as the vertex;
   - has $x = h$ as the axis of symmetry;
   - has $k$ as a minimum value if $a > 0$;
   - has $k$ as a maximum value if $a < 0$.

4. The vertex of the graph of $f(x) = ax^2 + bx + c$ is
   
   $$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right),$$

   where $\frac{-b}{2a}$ is the $x$ coordinate, and $f\left(\frac{-b}{2a}\right)$ is the $y$ coordinate.

5. Absolute value equations:
   
   $$|X| = a \iff X = a \text{ or } X = -a$$

6. Absolute value inequalities:
   
   $$|X| < a \iff -a < X < a$$
   $$|X| > a \iff X < -a \text{ or } X > a$$
Test 4 (4.1-4.4)

(1) Leading term test
If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ or as $x \to -\infty$ can be described in one of the four following ways:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a_n &gt; 0$</th>
<th>$a_n &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Odd</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>

The portion of the graph is not determined by this test.

(2) Intermediate Value Theorem:
If $f(a)$ and $f(b)$ have opposite signs, then there is at least one real zero between $a$ and $b$.

(3) Dividing $f(x)$ by a factor $(x - c)$ yields a quotient $Q(x)$ and a remainder $R$.
This can be summarized as:

$$f(x) = (x - c)Q(x) + R$$

(4) Remainder theorem: $f(c) = R$

(5) Factor theorem: if $R = 0$, then $(x - c)$ is a factor of $f(x)$

(6) Factors and zeros: if $(x - c)$ is a factor, $c$ is a zero, and vice versa

(7) Rational zeros theorem:
Let

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0,$$

where all the coefficients are integers. Consider a rational number denoted by $p/q$, where $p$ and $q$ are relatively prime (having no common factor besides -1 and 1). If $p/q$ is a zero of $P(x)$, then $p$ is a factor of $a_0$ and $q$ is a factor of $a_n$.

(8) Descartes’ rule of signs:
Let $P(x)$ written in descending or ascending order be a polynomial function with real coefficients and a nonzero constant term. The number of positive real zeros of $P(x)$ is either:

(a) The same as the number of variations of sign in $P(x)$, or

(b) Less than the number of variations of sign in $P(x)$ by a positive even integer.

The number of negative real zeros of $P(x)$ is either:

(a) The same as the number of variations of sign in $P(-x)$, or

(b) Less than the number of variations of sign in $P(-x)$ by a positive even integer.

A zero of multiplicity $m$ must be counted $m$ times.

Test 5 (5.1-5.5)

(1) Definition of log: $\log_a x = y \leftrightarrow a^y = x$

(2) Product rule: $\log M + \log N = \log (MN)$

(3) Quotient rule: $\log M - \log N = \log \left( \frac{M}{N} \right)$

(4) Power rule: $\log M^p = p \log M$

(5) Change of base formula: $\log_b a = \frac{\log_a a}{\log_a b}$
(6) General log facts

- \( \log(x) \) means \( \log_{10} x \)
- \( \log_a 1 = 0 \)
- \( \log_a a = 1 \)
- \( \log_a a^x = x \)
- \( a^{\log_a x} = x \)

• Last Section: 5.6

(1) Compound interest formula: \( A(t) = A_0(1 + \frac{1}{n})^{nt} \)

(2) Continuously compounded interest formula: \( P(t) = P_0e^{kt} \)

(3) Exponential growth formula: \( P(t) = P_0e^{kt} \)

(4) Exponential decay formula: \( P(t) = P_0e^{-kt} \)

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**A Library of Graphs**

**Linear Graphs:** \( f(x) = mx + b \)

\[ f(x) = \frac{2}{3}x - 2 \]

\[ f(x) = \frac{2}{3}x - 2 \]

**Vertical Line**

\[ x = 3 \]

**Horizontal Line/ Constant Function**

\[ f(x) = -2 \]

**Quadratics:** \( f(x) = ax^2 + bx + c \) or \( a(x-h)^2 + k, a \neq 0 \)

\[ f(x) = x^2 \]

\[ f(x) = 2x^2 - 3x - 1 \]

\[ f(x) = -(x+2)^2 + 2 \]
Cubics: $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$

- $f(x) = x^3$
- $f(x) = x^3 + 2$
- $f(x) = x^3 - 2x^2 - 4x + 3$

Exponential: $f(x) = a^x$, $a > 0$  and  Logarithmic: $f(x) = \log_a(x)$

- $f(x) = e^x$
- $f(x) = 3^x$
- $f(x) = \frac{1}{2^x}$
- $f(x) = 4^{-x}$
- $f(x) = \log_{10}(x)$
- $f(x) = \log_e(x)$

Miscellaneous:

- Rational Function $f(x) = \frac{2}{x}$
- Square Root Function $f(x) = \sqrt{x}$, $x \geq 0$