Practice Test 1

Name:

Q.1) A strain of bacteria reproduces asexually every 50 minutes. That is, every 50 minutes, each bacterial cell splits into two cells. If, initially, there are 10 bacteria, how long will it take until there are 640 bacteria?

Q.2) Assume that the population growth is described by the Beverton-Holt recruitment curve with growth parameter $R$ and carrying capacity $K$. Find the population sizes at $t = 1, 2, ..., 5$ for the given initial value $N_0$.

$$R = 2, K = 10, N_0 = 2$$

Q.3) Compute $N_t$ and $N_t/N_{t-1}$ for $t = 2, 3, 4, ..., 15$ when

$$N_{t+1} = N_t + N_{t-1},$$

with $N_0 = 1$ and $N_1 = 1$.

Q.4) Use the limit laws to evaluate each limit.

a) $\lim_{x \to -1} (x^3 + 7x - 1)$

b) $\lim_{x \to 1} \frac{(x-1)^2}{x^2-1}$

Q.5) Let

$$f(x) = \begin{cases} \frac{x^2-x-6}{x-3} & \text{if } x \neq 3 \\ \frac{a}{3} & \text{if } x = 3 \end{cases}$$

and find $a$ so that $f(x)$ is continuous at $x = 3$. 
Q.6) Determine whether $f(x)$ is continuous at given points.
   a) $f(x) = x^3 - 2x + 1$, $c = 2$
   b) $f(x) = \begin{cases} \frac{1}{x} & \text{for } x \geq 1 \\ \frac{1}{2x} & \text{for } x < 1 \end{cases}$, $c = 1$

Q.7) Suppose that the size of a population at time $t$ is given by
   
   \[ N(t) = \frac{100}{1 + 9e^{-t}} \]

   for $t \geq 0$. Determine the size of the population as $t \to \infty$, using basic rules for limits.

Q.8) Evaluate the limits
   a) $\lim_{x \to \infty} \frac{1-x^3+2x^4}{2x^2+x^4}$
   b) $\lim_{x \to \infty} \frac{3e^2x}{2e^{2x} - e^x}$
   c) $\lim_{x \to 0} \frac{\sin(\pi x)}{x}$

Q.9) Let
   
   \[ f(x) = x^2 - 1, \quad 0 \leq x \leq 2. \]

   Use the intermediate theorem to conclude that there exists a number $c \in (0, 2)$ such that $f(c) = 0$.

Q.10) The following function describes the height of a tree as a function of age:
   
   \[ f(x) = 132e^{-20/x}, \quad x \geq 0. \]

   Find $\lim_{x \to \infty} f(x)$. 