Practice Test 3

Name:

Q.1)(10 points) Find the critical points for the following functions
a) \( f(x) = 3x - x^2, \ x \in \mathbb{R}, \)

b) \( f(x) = -\frac{2}{3}x^3 + \frac{7}{2}x^2 - 3x + 4, \ x \in \mathbb{R}. \)

Q.2) (5 points) Find the point of inflection for the following function
\[
y = \frac{2}{3}x^3 - 2x^2 - 6x + 2, \quad x \in \mathbb{R}.
\]
Q.3) (7 points) Find the local maxima and minima of the following function

\[ x^4 - 2x^2, \quad x \in \mathbb{R}. \]

Q.4) (7 points) Determine whether the given function have absolute maxima and minima and if so find their co-ordinates

\[ y = (x - 1)^3 + 1, \quad x \in \mathbb{R}. \]

Q.5) (7 points) For the given function, find the intervals on which it is increasing and the intervals on which it is decreasing

\[ y = (2 - x)^2, \quad -2 \leq x \leq 3. \]
Q.6) (7 points) For the given function, find the interval on which it is concave up, and on which it is concave down

\[ y = \frac{2}{3}x^3 - 2x^2 - 6x + 2, \quad -2 \leq x \leq 5. \]

Q.7) (7 points) Consider the chemical reaction

\[ A + B \rightarrow AB. \]

Suppose that reaction rate for the given reaction is given by the following function

\[ R(x) = k(a - x)(b - x), 0 \leq x \leq \min(a, b), \]
where $x$ is the concentration of the product $AB$ and $\min(a, b)$ denotes the minimum of the two values of $a$ and $b$. The constants $a$ and $b$ are the concentrations of the reactants A and B at the beginning of the reaction. Find the concentration $x$ that maximizes the reaction rate for the following constants

$$k = 2, a = 4, b = 5.$$ 

Q.8) (15 points) A field biologist wants to enclose a rectangular study plot. She has 1600 ft of fencing. Using this fencing, determine the dimensions of the study plot that will have the largest area.
Q.9) (15 points) Find the smallest perimeter possible for a rectangle whose area is $25in^2$.

Q.10) (10 points) Find the largest possible area of a right triangle whose hypotenuse is $4cm$ long.