(1) (Serge Lang p93/1) Let \( A_1, \cdots, A_r \) be vectors in \( \mathbb{R}^n \). Let \( W \) be the set of vectors \( B \) in \( \mathbb{R}^n \) such that \( B \cdot A_i = 0 \) for every \( i = 1, \cdots, r \). Show that \( W \) is a subspace of \( \mathbb{R}^n \).

**Solution:** Let \( B_1, B_2 \in W \). Let \( \alpha, \beta \) be numbers. Then \( B_1 \cdot A_i = 0 \) and \( B_2 \cdot A_i = 0 \) for every \( i = 1, \cdots, r \). For every \( i = 1, \cdots, r \),
\[
(\alpha B_1 + \beta B_2) \cdot A_i = \alpha B_1 \cdot A_i + \beta B_2 \cdot A_i = 0.
\]
So, \( \alpha B_1 + \beta B_2 \in W \) and hence \( W \) is a subspace.

(2) (Serge Lang p93/5) Let \( V \) be a subspace of \( \mathbb{R}^n \). Let \( W \) be the set of elements of \( \mathbb{R}^n \) which are perpendicular to every element of \( V \). Show that \( W \) is a subspace of \( \mathbb{R}^n \). This subspace \( W \) is often denoted by \( V^\perp \), and is called \( V \) perp, or also the **orthogonal complement of** \( V \).

**Solution:** Let \( w_1, w_2 \in W \) and \( \alpha, \beta \) be two numbers. Then \( w_1 \cdot v = 0 \) and \( w_2 \cdot v = 0 \) for any \( v \in V \). For any vector \( v \in V \),
\[
(\alpha w_1 + \beta w_2) \cdot v = \alpha w_1 \cdot v + \beta w_2
\]
\[
= 0.
\]
So, \( \alpha w_1 + \beta w_2 \in W \) and hence \( W \) is a subspace.