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SPATIALLY-VARYING COMPACT MULTI-POINT FLUX APPROXIMATIONS FOR 3-D ADAPTED GRIDS WITH GUARANTEED MONOTONICITY

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ABSTRACT

We propose a new single-phase local transmissibility upscaling method for adapted grids in 3-D domains that uses spatially varying and compact multi-point flux approximations (MPFA). The multi-point stencils used to calculate the fluxes across coarse grid cell faces are required to honor three generic flow problems as closely as possible while maximizing compactness. We also present a corrector method that adapts the stencils locally to guarantee that the resulting pressure matrix is an M-matrix. Finally, we show how the computed MPFA can be used to guide adaptivity of the grid.

Introduction

In this work we are concerned with transmissibility upscaling for coarse-scale modeling of subsurface formations with complex heterogeneity. Designing a transmissibility upscaling method that is both accurate and computationally efficient is challenging. Upscaling methods that use two-point flux approximations (TPFA) may not give sufficiently accurate upscaling results in formations with large scale connective paths that introduce full-tensor anisotropy at coarse scales [1, 2]. On the other hand, MPFA methods, which involve additional cells, add computational costs and may suffer from non-monotonicity [3].

Our goal is to construct a local multi-point flux approximation that accommodates full-tensor anisotropy and guarantees a monotone pressure solution. To achieve this we allow the MPFA stencil to vary spatially, we minimize the number of points involved in the stencil, and we let the stencil revert to a TPFA stencil where this is sufficiently accurate. We exploit the added freedom in allowing the stencils to vary from grid cell to grid cell in order to guarantee monotonicity. Our method, introduced in [4], is called the Variable Compact Multi-Point method, or VCMP.

VCMP has been implemented in two spatial dimensions on various types of grids with cell-centered finite volume discretizations. In this paper, we describe an extension to 3-D problems, and how VCMP can guide adaptive mesh refinement.

Fine and Coarse Grid Equations

The upscaling strategy is based on single phase, steady and incompressible flow. The governing dimensionless pressure equation is $\nabla \cdot (k \cdot \nabla p) = 0$. Here p is the pressure and k the permeability tensor.We follow the common practice of using this same equation for the coarse pressure, which is justified in [5].

Construction of VCMP for Cartesian Grids

We aim to construct a multi-point finite-volume scheme that is “close” to a two-point scheme, for efficiency and robustness. However, it should be very accurate for smooth pressure fields, even in cases of full-tensor anisotropy. If the pressure field is not smooth, improved accuracy can be achieved by local grid refinement. Therefore, the scheme should be applicable to adaptive grid strategies, such as the CCAR strategy developed in [1, 6].

To create a MPFA with these properties, we allow the stencil

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to vary per cell face. Our MPFA uses a subset of the ten pressure values $p_j,\ j = 1,\ldots,10$ surrounding the face. Odd-numbered and even-numbered points are on opposite sides of the face, with points 1 and 2 being the points used in a TPFA, and the remaining points chosen to be the centers of neighboring cells. For each j, we let t_j denote the weight that will be assigned to point j in the flux approximation, which has the general form $f = -\mathbf{t}^T\mathbf{p}$, where $\mathbf{t} = \left[\, t_1 \cdots t_{10} \,\right]^T$, and $\mathbf{p} = \left[\, p_1 \cdots p_{10} \,\right]^T$.

We solve the pressure equation on the local region of the fine grid containing the ten points with three generic Dirichlet boundary conditions. We let $\mathbf{p}^i(x,y)$, $i = 1,2,3$, be the solutions of these local problems, and p^i_j denote the value of $\mathbf{p}^i(x,y)$ at point j. The pressure field $\mathbf{p}^1$ is computed using boundary values chosen so that the pressure gradient is across the face, and $\mathbf{p}^2$ and $\mathbf{p}^3$ are obtained from boundary values chosen so that the pressure gradient is parallel to the face.

For $i = 1,2,3$, we let f_i denote the coarse-scale flux (sum of fine-scale fluxes) across the face obtained from the local solution $\mathbf{p}^i(x,y)$. To compute the weights t_j, we solve the constrained optimization problem

$$\min_{\mathbf{t}} \sum_{i=1}^{3} \alpha_i^2 \left|\mathbf{t}^T\mathbf{p}^i + f_i\right|^2 + \sum_{j=3}^{10} \beta_j^2 t_j^2, \tag{1}$$

$$\sum_{j=1}^{10} t_j = 0, \quad t_{2j-1} \le 0, \quad t_{2j} \ge 0, \quad j = 1,\ldots,5. \tag{2}$$

In the current implementation, the weights α_i are chosen to be $|f_i|$ and the weights β_j are chosen to be equal to $(|f_1| + |f_2| + |f_3|)/M$, where M is a tuning parameter. The larger the value of M, the more closely the flows are honored.

The M-fix for Guaranteeing Monotonicity

While VCMP is robust, it does not guarantee an M-matrix. This property is highly desirable, as it ensures monotonicity of the pressure solution, and improves solver efficiency [7]. We therefore employ the M-fix, a corrector method introduced in [8] for the 2-D case. It entails identifying matrix entries with the wrong sign, and recomputing the corresponding MPFAs, with additional constraints chosen in such a way as to guarantee an M-matrix. If the modified optimization problem is infeasible, which rarely occurs in practice, a two-point flux is used instead.

Application to Adaptive Mesh Refinement

In [1], a grid adaptation strategy was introduced in which cells surrounding a face are refined if, in global coarse-scale flow simulations, a sufficiently large fraction of the total flow passes through the face. However, this causes unnecessary refinement when flow is oriented with the grid. We solve this problem by refining only if the weights t_3-t_{10} are sufficiently large, which only occurs in the presence of full-tensor anisotropy. In addition, as an alternative to applying the M-fix, we can refine where elements of the matrix have the wrong sign, or where no stencil that satisfies the sign constraints (2) can be computed by VCMP.

Summary and Conclusions

We have generalized VCMP to 3-D domains. VCMP accommodates full-tensor anisotropy, which is generally present in coarse-scale flow problems. The stencil adapts to the orientation of the underlying fine permeability distribution, and can be used as an indicator for adaptivity.

Lack of monotonicity may occur in cases where strong permeability contrasts are not aligned with the grid. As in the 2-D case, the M-fix can be used as a corrector step in the VCMP method to guarantee the pressure matrix is an M-matrix.

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