

THE LUHN ALGORITHM

The Luhn algorithm is used to verify that a credit card number is valid.

- (1) Multiply every even-position digit (from right) by two.
 - (a) If a product is greater than ten, add the digits.
- (2) Add the numbers in the even positions.
- (3) Add the numbers in the odd positions, *except* the rightmost digit.
- (4) Add these last two values together.
- (5) The rightmost digit should be what it takes to get the sum to be a multiple of 10.

You may want to try it with one of your own credit cards first, just to make sure it works.

Suppose a company wants to release a new, 12-digit card.¹

- (1) If the first 11 digits are 70331290121, what must the 12th digit be?
- (2) Show that the Luhn algorithm detects every error where one number changes.
- (3) Show that the Luhn algorithm detects *almost* every error of one transposition (e.g., 7303... instead of 7033...).
- (4) Which transpositions can the Luhn algorithm not detect? Give an example from the number above.

See what happens when you transpose two numbers.

¹Most credit cards are 16 digits long. American Express is 17 digits long. To my knowledge, no bank releases a 12-digit card.