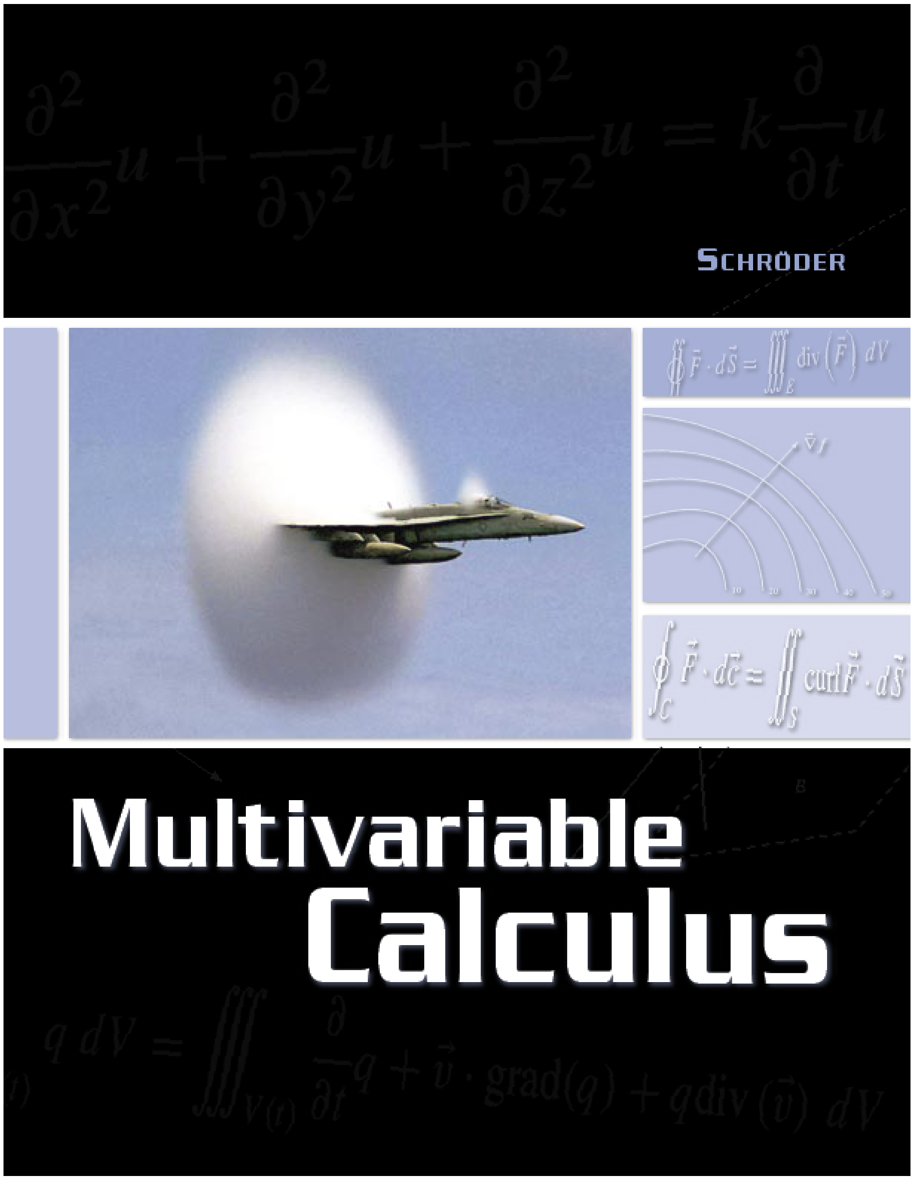

compiled: September 14, 2005

This is the table of contents for the final text on multivariable calculus and differential equations. Multivariable calculus is available from Fountainhead. The full text can be printed immediately on request, but I want to edit the differential equations once more.

Multivariable Calculus and Differential Equations

Bernd S. W. Schröder

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Foreword

The foreword is the only place in the text where the word "I" is used. I want the foreword to be a place in which you, the reader, and I can have a fictional conversation about the overall ideas that guide this text. In this "conversation" I will point out various features and give advice on how to be successful in mathematics and in college in general. In the text itself, mathematics is front and center.

Overall, this foreword is a set of ideas that I have collected from colleagues, students and literature over the years. It could have been presented by me in a typical first day of class presentation. Many ideas have been refined through discussions with colleagues and students. Thus teachers and students alike should find the ideas reasonable. I claim no credit for any of them, because, as I said, they all originated some time through conversations, observations, etc.

You are free to take or leave any of the advice in here. Some statements may not apply to you because you already do what is recommended. But, consider everything at least once.

Symbols In The Text

The foreword starts with some special symbols that I use. That way features encoded with special references will not remain hidden, even if you don't like reading whole forewords.

- The present text is part of an overall set of modules that concisely and consistently covers freshman and sophomore mathematics as required by engineering, science and mathematics majors. Content goes from precalculus through single variable and multivariable calculus up to differential equations and some statistics. The overall concept is modular, but numerous cross connections are used to (re)emphasize important topics.

  To take the mystery out of cross connections to another volume, references to results in sections that are not included are indicated in (parentheses). A brief synopsis of these references can be found in Appendix D which starts on page 625. For example, (DUM) refers to the explanation marked (DUM) on page 628. Modules are referred to in boldface. Sections, theorems, etc. are referred to in regular type, such as DUM.A.

- Exercises to which a hint or a solution is given in the back are marked with numbers on darker background. For example, an exercise numbered #. indicates that there is a hint or a solution in Appendix E starting on page 639.

  This is a break with the unhealthy tradition of having answers to every second problem (usually the odd-numbered ones) in the back of the book. In freshman and sophomore mathematics, future professionals are trained. There is no profession in which the answer to every second problem can be looked up anywhere. In the early years of college, the textbook oracle that says "yay" or "nay" must be replaced with the ability to devise other methods to check a solution. These methods include estimation, plugging solutions into the original equations, differentiating an integral etc.

  For problems for which I found it reasonable, a solution or a hint are included. For sets of problems that train the same skill, only one hint or solution is necessary to get started.

  It should be noted that a complete solutions manual to facilitate instruction is in preparation. This solutions manual is not intended for public release however. Students who excessively depend on solutions manuals shortchange themselves.

To The Student

Also read the part that is addressed to the teacher. The more you understand the reasons why things are taught in a certain way, the easier it will be to learn what is taught.

Congratulations! Having made it through single variable calculus, you are now ready to tackle problems of near arbitrary complexity. Multivariable calculus is a place where mathematics truly meets real life. With multivariable calculus we can describe three dimensional (that is, real) phenomena. All suggestions and recommendations from the single variable text still apply. I will briefly add a few and emphasize some earlier ones.

Focus on the concepts and do not shy away from computation. At the end of modeling a problem, scientists and engineers often say that the rest is "just mathematics". Usually, after one or several pages of computation you realize that, yes, it was just mathematics, but there was a lot of it. The beauty of applying mathematics to a real life situation is *that* it allows us to model and predict real life via computations. The computations can be complicated, but still they are merely computations. There is no magic required and there is no uncertainty. To meet the challenge thus presented, we must understand the concepts so that we can get from reality to the "just mathematics" stage, and then we must be able to execute the necessary computations.

Because of the wide variety of problems that can be solved with the mathematics presented in this text, there is a near unlimited number of formulas that one could derive. I have purposely limited the number of formulas presented in the text. This is because a smaller number of formulas better showcases the concepts and it also reinforces the use of fundamentals. Solid fundamentals plus conceptual understanding will get you through situations which are unreachable just with formulas. Usually, examples in the text are solved with a "minimum number of memorized formulas" approach.

Don't just memorize words and symbols. Internalize meaning.

Effective and useful memorization is a necessary and useful cognitive process. I have never been a proponent of excessive memorization, but I will also never discount its value. Certain facts simply have to be memorized. There is no alternative to the need to remember. There are several ways to get there, though. To derive the intended benefit from memorization, the facts to be memorized must be connected to their uses in a cognitive way. In this fashion, they can be activated when they are needed.

For example, to memorize the amendments to the constitution of the United States, it helps to put them in a historical context. The amendments are in chronological order. While the dates are not stated, approximate dates can be inferred from the statements themselves. This leads to some thoughts about why the amendment may have come to be. In this fashion, repeated slow reading of the amendments plus some layman type analysis leads to their memorization.

Similarly, in this text there are many formulas. If we understand the components of the formula and what the formula does, the memorization is a snap. This is another reason why formulas are put into applied contexts.

In cases where formulas for special cases may help, these formulas are usually turned into exercises. Feel free to use the results stated in these exercises as needed. A typical example is working with surfaces in three dimensional space. All concepts must be stated using parametric surfaces, because arbitrary surfaces can only be modeled parametrically. Formulas for the special case of surfaces $z = f(x, y)$ exist, but I turned them into exercises. These exercises will train you to work on a slightly more abstract level than direct computations. If you function better using these formulas (in many cases they are all that is needed), then use them. But remember, there will be situations where working with the general formulas in terms of parametric surfaces cannot be avoided.

Do not be discouraged if reading goes slowly. Mathematics is meant to be read slowly. In multivariable calculus the concepts get deeper and the computations get longer. Visualizations will involve space, time and also data as appropriate. Computationally, what used to be a problem all by itself in single variable calculus now becomes just a part of a larger computation. The simplest observation of this fact is that where we had only one function to contend with in single variable calculus, in multivariable calculus functions will come in twos or threes. One for each coordinate of two or three dimensional space.

To put it in a nutshell, though the information density in single variable calculus already is high, it increases once more as we go beyond single variable calculus. Consequently, we need to read even more thoroughly, we need to carefully make connections between the various concepts and we need to connect the mathematics to real life visualizations. To this end, there is a larger than usual number of cross references to explanations as well as computations in other modules. These cross references are intended to make you revisit fundamental ideas and to make explicit mental connections between known fundamentals and the new content. At the same time they will make your reading pace a bit more deliberate.

Keep using the margins for side calculations. Computations will be lengthy at times and often a condensed version will be presented. Use the margin to fill in skipped steps, etc. Also, if a computation is interrupted with a reference to a computation in an earlier example, it is a good exercise to reproduce the computation before looking it up. Using the margins for computations and remarks on how to visualize a concept will allow you a deeper connection to the material you read.

The ultimate goal is to give you the ability to "see" the reality behind your computation as you are doing the computation. Though it may be challenging at times, the connection between mathematics and reality will become very "real" to you are you go through this text. To function competently as a scientist or engineer, it is best to not have any barriers between mathematics and an application. Mathematics helps solve the applications and the applications also guide the mathematical solution.

The mathematician D. Hilbert once said, "Man muß jederzeit anstatt Punkt, Gerade, Ebene auch Tische, Stühle, Bierseidel sagen können". Loosely translated, you should always be able to replace points, lines and planes with tables, chairs and beer steins. The idea is not to start drinking. Rather, the idea is to use objects from your everyday experience to visualize the abstractions and computations of mathematics and theoretical science. The fanciest four color graphics and animations will pale in comparison to a simple visualization that you found and that works.

To The Teacher

Let's start with the most important statement. Nothing in this text is a prescription how you should teach your classes. You will have methods that I am absolutely unaware of and which work beautifully. None of these ideas should be undermined by any textbook. If anyone remarks negatively on deviations from this text, please refer them to this section. I have never been arrogant enough to assume I have all the answers. Moreover, even if I had all the answers, I would be realistic enough to realize that I cannot enforce them like a cookie cutter on all the teachers and students in the world. That being said, I hope that you will find this text a valuable resource.

Also read the part that is addressed to the student. Several ideas are explained that can stand some classroom reinforcement.

After single variable calculus, nearly any topic can be covered. Figure 1 shows the logical structure that underlies the modular materials. It shows that single variable calculus really is the core after which multivariable calculus, differential equations and statistics can be tackled in almost any order. This structure gives freedom to shift content in order to better serve concurrent classes in science and engineering and to avoid problems with conceptual "spikes" as shown in Figure 2. The structure also reveals an introduction to partial differential equations as a natural culmination of the calculus experience. Of course this is too large a topic to treat even in a separate year long course. However, many fundamental partial differential equations can be introduced in the context of vector calculus. This

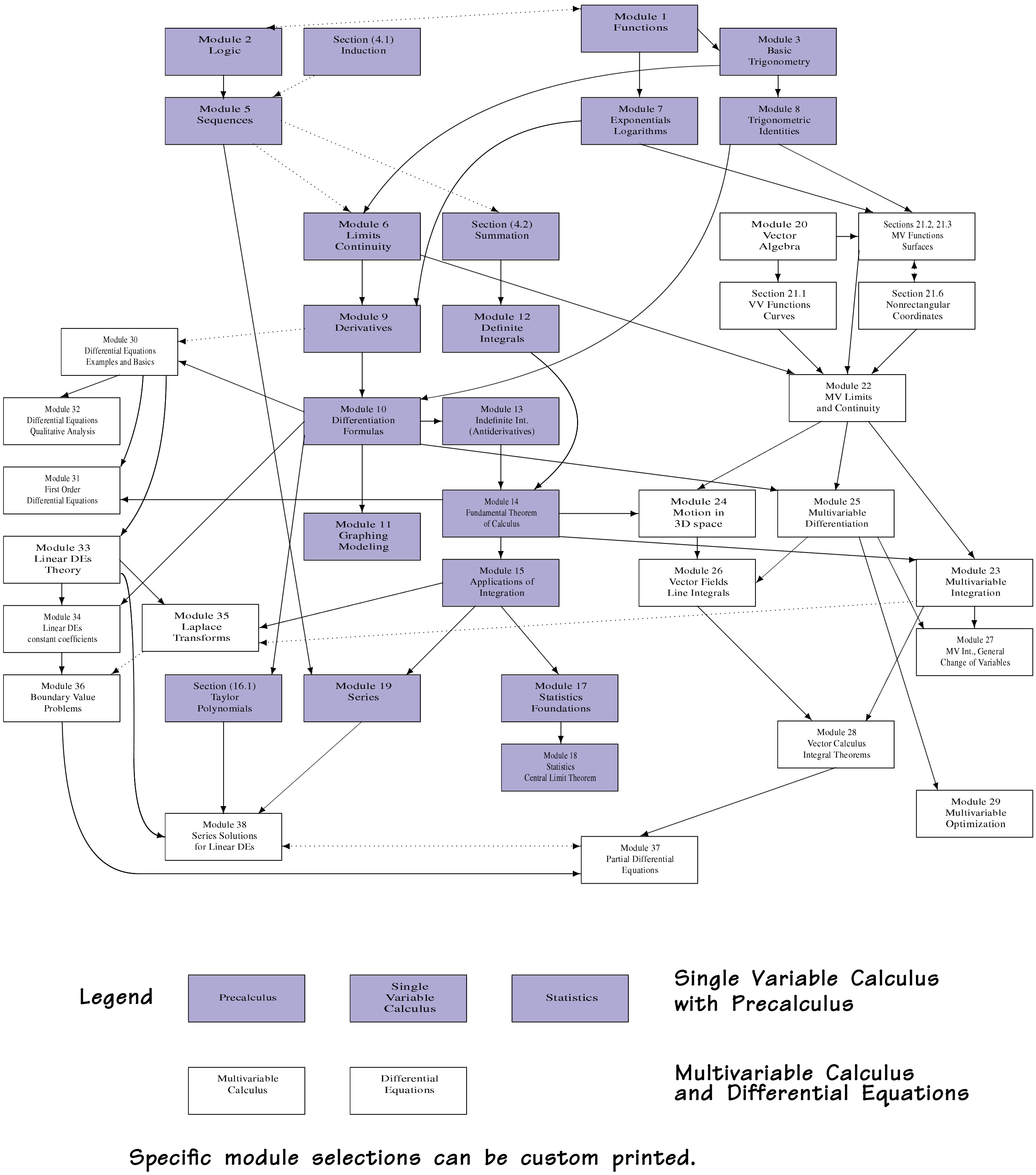

Figure 1: Flowchart showing the logical dependence of the modular materials that make up the texts. Prerequisites are indicated by solid arrows. Dotted arrows indicate topics that reinforce each other or prerequisites that can possibly be omitted. The white boxes mark the modules included in this text.

emphasizes the natural connections between mathematics, science and engineering when Gauss' and Stokes' Theorems are connected to electrodynamics, heat transfer and fluid dynamics as done in Module 28. Exposure to some basics as in Module 37 also trains symbolic work with partial derivatives.

Accessible proofs are presented whenever possible. Proofs are the engine that drives mathematics. Thus omitting proofs would be like selling a car without an engine. At the same time, use your best judgment as to how much you want to work with the engine. Many people drive without being able to fix the engine. Then again, some people go far because they are adept at working with the engine. (End of metaphor.) Most proofs are quite computational, but that is the nature of the subject. Students should be reassured that it takes time to develop the ability to produce proofs and that very good science can be done without going all the way in the theory.

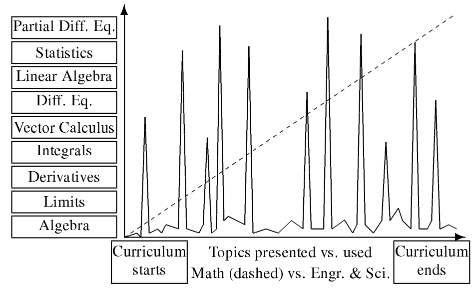

Figure 2: Comparison of the use of mathematics in other disciplines to the order in which it is usually presented. The horizontal axis represents time in a curriculum, the vertical axis represents mathematical topics in a typical order. The dashed line presents the progression through topics in mathematics, the solid graph represents the use of topics in the applied disciplines. In the applied disciplines much of the work is algebra, complemented by occasional conceptual "spikes" into more sophisticated topics such as integration, vector calculus, statistics or (partial) differential equations.

Pathological examples, if they exist, are usually presented at the end of the respective sections. Pathological examples are like extreme driving conditions. We don't encounter them often, which means we should spend most of our time preparing for situations we can realistically expect to encounter. (My experience in driving on icy mountain roads does not help at all when navigating city traffic.) At the same time, we must be aware of pathological situations. Hence we cannot pretend pathological situations never occur. A concern of scientists and engineers is that mathematics courses can focus too much on technical details, rather than preparing students for computations common in science and engineering where functions usually have all the smoothness conditions we need. My solution was to discuss potential pathology (multivariable differentiation and nonorientable surfaces immediately come to mind) at the end of the respective sections after the groundwork for day-to-day science, engineering as well as mathematics has been laid.

About the Cover

Multivariable calculus is the course in which a student breaks through the final barriers between early mathematical and scientific preparation and more advanced content such as statics, dynamics, field theory (electricity, magnetism, gravity), fluid dynamics, heat transfer and diffusion phenomena, optimization of complex processes, etc. Connections to these fields are made throughout this text. Though we will not work on details of supersonic flight, the jet gives a visual impression of breaking these barriers. The figures to the right are the Divergence Theorem (cf. Theorem 28.2.3) at the top, a visualization of the gradient as direction of steepest ascent (cf. Figure 25.25) in the middle, and Stokes' Theorem (cf. Theorem 28.3.4) at the bottom. These ideas are part of vector calculus (cf. Module 28), which is the pinnacle of any calculus course and the connection to most of the mentioned applied topics. A student with a firm grasp on vector calculus should be well prepared for advanced work in engineering and the sciences. The modeling in vector calculus trains all the necessary visualization and modeling, while the computations are a built-in review of most of the skills acquired in the student's earlier mathematical career.

Acknowledgements

This work has grown over several years and many colleagues have given me valuable input on the text or made me aware of tools I could use.

Khaled Al-Agha and Galen Turner proofread early versions of some modules. Typos and suggestions were reported by many colleagues, most notably (so far) John Hunt, Jinko Kanno, Jim Marion, Dave Meng, Nathan Ponder and Natalia Zotov. Danny Eddy helped take pictures connected to chemistry and Pedro de Rosa helped take pictures connected to physics. Special thanks are due to my academic director, Gene Callens, who has been a great mentor over the years.

Numerous students have provided input through telling me about what is needed in their upper level classes as well as by reporting typos. To all these students I am grateful.

Since Fall 1998, Devery Rowland printed numerous early versions of a variety of modules. Through our conversations he also taught me a lot about about the production process and in-house printing possibilities. Ann White and her staff at the Tech book store were very accommodating in the dissemination of locally produced materials.

Funding was provided in part by the Louisiana Board of Regents under a "Fellow of Excellence in Engineering Education" grant and by the National Science Foundation under Action Agenda grant AA9972729 "Institutionalization of an Integrated Engineering Curriculum at Louisiana Tech University" and the NSF-CCLI A&I grant nr. 0311481 "Integrating the Sciences and Secondary Science Education in the Early College Curriculum." Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation or other funding agencies or acknowledged persons or entities.

Some homework problems have been extracted from an exam database made available by Kansas State University's Department of Mathematics.

Sources of pictures not produced by the author are provided in the caption.

Felix Frazier, Amy Chapman and everyone at Fountainhead Press are a pleasure to work with and they have been highly supportive throughout the development of this project. Moreover, Fountainhead's philosophy of high quality at a reasonable price with no unnecessary add-ons exactly fits my own idea of what textbooks should be.

For all the above support I am deeply grateful. However, to no one am I more indebted than to my family. My wife, Claire, and my children, Samantha, Nicole, Haven and Mlle have seen me vanish into the attic time and again to pursue my various mathematical projects. A good bit of work was once more done in the basement of the house of my parents in law, Jean and Merle. Finally, my mother, Gerda, and my father, Siegfried, gave me the foundation on which my professional career is built. Without their love and support this work would not have been possible.

Ruston, LA, September 14, 2005

Bernd Schröder

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