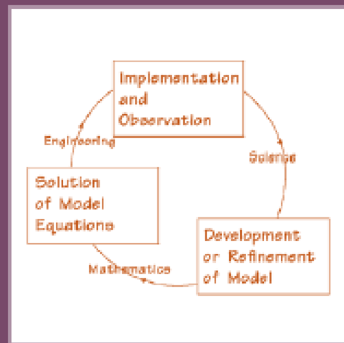
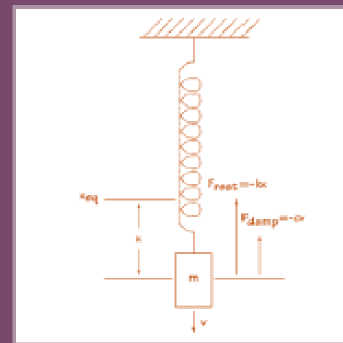
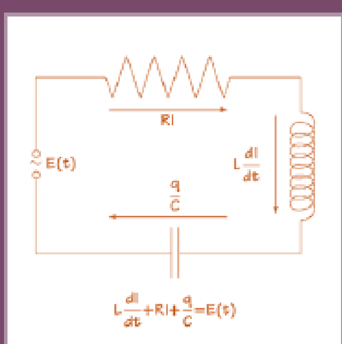
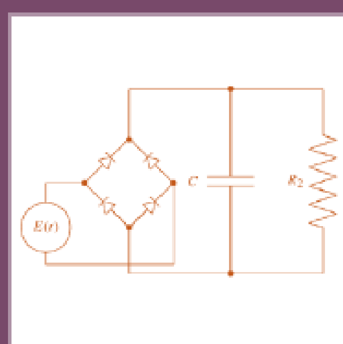
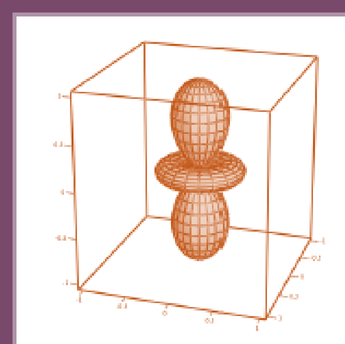




A Workbook for DIFFERENTIAL EQUATIONS







BERND S.W. SCHRÖDER

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Contents

Preface

A differential equations course at the level of this text is taken mostly by engineering, science and mathematics majors. So you and your fellow students have similar goals and interests. The examples and topics in this text have been chosen to serve these interests. But ultimately, the specifics of *your* discipline define how you will apply the course content. Therefore, it is a good idea to find out from a professor in your discipline which parts of this course will be most important to you. On the other hand, do not be discouraged by examples that are not in your field or if you are not majoring in engineering, science or mathematics. The text is accessible from calculus.

The main premise is that **differential equations are motivated by applications.** Your discipline requires the study of differential equations because they are used to model and predict reality. Therefore, the overriding concerns are to showcase the uses of differential equations and to provide computational and conceptual tools to investigate and apply them. When applications are investigated, engineering, science and mathematics merge. Consequently, the challenges posed by the individual disciplines are magnified. To overcome these challenges, the following describes how to study and use differential equations (and possibly mathematics, science and engineering in general).

Productive engineers, scientists and mathematicians use a deep understanding of the "big ideas" together with good computational abilities to produce results. That is, to be effective, they do not clutter their minds with unnecessary details. It may be surprising, but the number of truly "big ideas" in engineering, science and mathematics is quite manageable. "Big ideas" include certain fundamental laws, such as Newton's laws, conservation laws and fundamental mathematical concepts, like the derivative and the integral. Computations connect these fundamental scientific ideas to specific applications. Although these computations may be quite lengthy, it is not necessary to recall every detail. In fact, it may even be counterproductive. Instead of memorizing details, we must build computational abilities so that we can complete long computations, given enough time. After all, the details will be different in the next application. *What stays with you are the fundamentals (the "big ideas") and your computational ability.* If you are not already working in this fashion, then a differential equations course is a good place to start. Almost every topic in this text is built on one big idea and the rest is computation. Recalling the big idea and then practicing the computations should lead to success. If you work like that, then you can separate fundamentals from details when an applied class gets technical. This separation can help you understand very sophisticated models. So, for the next three paragraphs, let us separately discuss fundamentals/applications, computation by hand and computational tools such as computer algebra systems.

You learn easier when you connect new material to applications you are familiar with. Conversely, unfamiliar applications can be a preview of content you will see later, and this prior exposure can help you choose appropriate mathematical methods. The first stage of learning is the simple retention of facts. Although it was indicated above that there are better approaches than memorizing too many details, certain details ("big ideas") *must* be recalled almost instinctively. Reliable recall is easier when you have a context to which new information can be related: In an experiment described in [4], chess players and individuals who did not play chess were shown arrangements of chess pieces on a board. Later, they were asked to reproduce the arrangement from memory. As long as the arrangements were random, both groups' performances were equal. If, however, the arrangements were meaningful in a game, the chess players' performance was measurably better than that of the control group. Experiments such as this one, as well as classroom observations, show that content that is related to existing knowledge is retained more easily than content that is learned in isolation. With respect to the applications in this text, there are two possibilities. If you are familiar with the application, then you want to use your familiarity to attach meaning to derivations and computations, so that the new content is retained more easily. If you are not familiar with the application, try to visualize it to gain physical insights. But also note that by connecting to this new application you may gain a head start on a class you may take later in your career. Once you have a firm grip on the "big idea," which can be an application or a key step for a computation, the details are "just computation."

"I have never met the guy who doesn't know how to multiply who created software ... Who has the most creative video games in the world? Japan! I never met these 'rote people'... You need to understand things in order to invent beyond them."

—Bill Gates

Good computational ability is an important foundation. You may have heard that the first course in differential equations requires a lot of lengthy, even tedious, computations. Plus, computations were also mentioned above. The bad news (if you will) is that this is true, because differential equations *are* a computational subject. Differential equations are important because they allow us to predict (read: *compute*) how real systems will behave. The good news therefore is not that the computations are always simple. The good news is that the computations are *possible*. For example, the computations used in the design of ready-to-fly aircraft without ever building a prototype are extremely detailed and tedious. But because they are possible, no effort needs to be spent on building and redesigning prototypes that will ultimately not be put into service. To show the full variety of computational difficulty that can arise even in simple models, I have included exercises ranging from quick computations to some that I would personally not touch without a computer. Your teacher will establish what you should do by hand and for what problems you can or should use a computer. *Independent of how long any assigned computations may seem, do them, do them carefully and, as much as possible, rely on (accurate) memory for facts from calculus.* Advanced texts in engineering, science and mathematics will expect that you can fill in "standard computations." It would be inefficient to use a computer algebra system for every little detail. In fact, dependency on a calculating device can lead to failure in advanced courses: If small details require too much conscious thought, then there is less brain space left to process the new content. (Now, I have no problem if you make it through relying on computational tools, but consider this: If you depend too much on a piece of silicon to be able to do your job, then a piece of silicon can replace you. I do not want that to happen to you.) This text can help you acquire the experience and the stubbornness that are needed in the rougher parts of an advanced course or text. Moreover, relying on (accurate) memory rather than references can also help improve your performance in this and future courses. By remembering facts from calculus (the "big ideas"), these facts are lodged more firmly in your memory, and repeated use (as opposed to repeated look-up) makes them more readily accessible in the future. Of course, when all else fails, look up facts from calculus while studying, but make an effort to not let them slip again.

Properly used computational tools will enhance your abilities. The fascinating thing about computations is that they can become almost arbitrarily difficult. Beyond a certain level of complexity, computers really are the only tool to complete certain computational tasks. The trick is to only use the computer for tasks that we cannot complete by hand. Many of the projects at the end of each module ask you to write a program for situations that you cannot (or may not want to) analyze by hand. The program in Project 6.7.1 will solve most homework problems in Chapter 6, and you can write this program as soon as you have completed Section 6.2! Speaking of computer projects, I have not included instructions for specific computer algebra systems. The goal of a mathematics text is to teach mathematics. Once the mathematics is understood, the skills needed to use a computer algebra system can be picked up from the documentation or by asking the right questions to a friend who is more versed with the program. Regarding Project 6.7.1, I should say that you should still do the assigned homework problems and then use the program to double check your results. In fact, that last part is worth repeating.

Check your results, frequently and carefully. The main problem in a course like this, as well as in life, is *not* that mistakes happen, but that they *sometimes remain undetected*. Mistakes are a basic part of life. That does not mean that faulty work, wrong decisions, etc. are acceptable at all. Quite the contrary, it means that we must safeguard against mistakes. For differential equations, this is easily done: Whatever solution you find, substitute it back into the equation and also check the initial or boundary values. If your solution satisfies both, then the problem is solved correctly. It's a simple step, but, if we want to address a very immediate concern you may have, it tells you *that* you solved an exam problem correctly before anyone grades it. To encourage you to check your own work, I have included double checks or reminders to double check in each module, and I have avoided the typical "every second answer provided" approach to the exercises. By verifying that solutions are correct without using the typical "oracle" that gives "the answer" in the back of the book, you will be better prepared for real applications, when there is no oracle. When an answer is provided, the problem number is boxed . But before you can check a solution, you need to find it, so let's get back to that.

The purpose of teaching is learning, and learning works best when learners teach themselves. Just about any pedagogical method has the same goal: To make you actively work with the content so that you retain the big ideas and internalize skills (like computations) that must be nearly automatic. That is, the goal is to get you to teach yourself. In mathematics, the first stage of actively engaging the content is to read the text. But reading is not always the same. Reading a newspaper article is slower and requires more conscious thought than reading a sound bite. Reading a short story is slower and requires more conscious thought than reading a newspaper article. Reading a novel is slower and requires more conscious thought than reading a short story. Reading mathematics and technical content (even if the piece you read is short) is slower and requires more conscious thought than reading a novel. In every step above, the density of information, the need to retain earlier information and the need to connect the content to existing knowledge increase. Reading technical content (such as a text on differential equations) requires you to methodically process information and to also fill in gaps in the presentation. As you read more technical texts, you will encounter more and larger gaps. Although these gaps can be frustrating, it's also quite a thrill to realize that you can fill them. Experienced readers of technical content fill the available white space with appropriate notes. These notes could be reminders where they have seen something similar before or computations that fill in skipped steps. In this fashion, when a passage is reread (yes, technical content typically is read and reread as often as necessary, until it is understood) fewer details will distract from understanding the "big idea" or from filling in every last gap. To model this behavior, in each module, handwritten notes are provided in the margin. At the beginning, the notes are what I would put into the margin as I read the printed part of the text. Later, the notes will just be reminders of what should be put in the margin, and, yes, I want you to fill in these notes. Near the end of the module, even the reminders will be left out. Make your own decisions of what to fill in. Being skilled at annotating a technical text goes a long way in engineering, science and mathematics, so use this as an opportunity for practice. (This is another reason why this text is a workbook. You are expected to put a certain amount of work into the margins.) Note that many figures are labeled in a handwriting font to indicate that a scientific reader will also create personal visualizations by drawing sketches. For an interesting take on active reading, consider [9]. Your own background will determine what kind of "reading practice" you need, which brings us to the next item.

A long time ago a mathematician said that "To read without a pencil is daydreaming." He was right then and he is still right today. (Also see the remarks on page 303.)

Knowing yourself can help you teach yourself. Your learning style determines a lot about your preferences in education. To find out more about learning styles and to learn more about your own learning style preferences, consider [6]. Knowing your learning style can help you understand why certain classes run or ran more or less smoothly for you. More importantly, the hints at [6] about what to do when your learning style does not match the subject might help you in classes in which the match is less than perfect. *After all, with all due respect to personal preferences, the overriding concern of any mathematical and scientific endeavor must be for nature's actual complexity.* The gap between human capabilities on one side and nature's actual beauty on the other is what makes learning, indeed life itself, a rewarding challenge. When the challenge seems to become too much, the insights from [3] might lift your spirits. It turns out that learning a subject in a "difficult" way is actually effective in the long run. In [3] it was demonstrated that skills learned under "difficult" conditions were retained better than skills learned under more optimal conditions. So my choices of sometimes hard examples and problems can be seen as part application (applications typically are hard) and part pedagogy, albeit of the "tough love" kind. With respect to this text, note that [27] can help if you learn better through lectures.

"Tell me and I forget, show me and I remember, involve me and I understand"? No one can involve me. I must involve myself. As soon as I do that, memory and understanding have a chance to grow. If I don't involve myself, someone will always tell me what to do.

Video available at [27].

A final word on the modular design. Solution methods for differential equations sometimes must be looked up quickly. This quick lookup can be discouraging when the needed method is buried deeply in a technical text. Typically it is not necessary to go through the whole text to understand one specific part. After reading Module 1, each module is accessible, except for Modules 7 and 9, which rely on earlier modules. Moreover, each module has specifically stated prerequisites and learning objectives. In this fashion, you can quickly determine what you need for a given subject and what the module will provide. For linear reading, Modules 1–4 give an overview of the fundamentals, before the remaining modules present theory, refinements and extensions. Overall, I hope this design helps you use the text efficiently, be it before you take a formal course on differential equations, say, as a resource in an applied course, or afterwards.

In summary, I hope you will enjoy reading this text as much as I enjoyed writing it.

Acknowledgements. Many individuals supported the development of this text. J. Carpenter, M. Countryman, D. Meng, N. Ponder and K. Zhong field tested early versions; N. Ponder improved the proof of Theorem A.4; K. Evans piloted the final version in a regular class; and D. Meng did the same in an on-line class. K. Evans and B. Camp developed and supported WeBWorK assignments for this course. J. Nelson provided much needed moral support from engineering. A. Saber proofread my attempt at an explanation for the boundary value problem of the bending beam. L. Sawyer helped with physics experiments, as did D. Cowling, S. Rovnyak and R. Szlavik with electrical engineering. N. Zotov helped by identifying instances in which I used the Kaiser's English, rather than the Queen's.

My student B. Bergeron inspired the treatment of suspended cables by asking a question regarding catenaries that came up in an actual engineering environment. A. Hector did an early version of the project in Section 6.7.5, which made us note the model's problem of "reflux" through the source. The original marginal notes were actually hand written, and J. Williamson helped scan in many of them. (Thank goodness I ultimately found a handwriting font. My handwriting is not exactly pleasant.) Many students pointed out typos in the text (please let me know if you find more, it can only make the text better).

D. Rowland tirelessly printed versions of these modules, and S. Green and later A. White and their staff at the bookstore remained patient in selling them. Susanne Steitz-Filler and her staff did their usual competent job as editors, and Lisa Van Horn assured that I would meet style and production parameters. (Any remaining problems with this text are my fault.)

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But the most important support of all is from my family. They are most affected by this kind of project, and without them none of this would have been possible or meaningful.

BERND S. W. SCHRÖDER

Ruston, Louisiana
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