Bernoulli Equations

Bernd Schröder

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That's it.

Solve the Initial Value Problem $y' + y = x^2 y^5$, $y(0) = 1$.

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$$n = 5, \qquad y = v^{\frac{1}{1-5}} = v^{-\frac{1}{4}}, \qquad y' = \frac{d}{dx} v^{-\frac{1}{4}}$$

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$$n = 5, \qquad y = v^{\frac{1}{1-5}} = v^{-\frac{1}{4}}, \qquad y' = \frac{d}{dx} v^{-\frac{1}{4}} \;\; = \;\; -\frac{1}{4} v^{-\frac{5}{4}} v'$$

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$$\begin{aligned} -\frac{1}{4} v^{-\frac{5}{4}} v' + \left(v^{-\frac{1}{4}}\right) &= x^2 \left(v^{-\frac{1}{4}}\right)^5 \\ -\frac{1}{4} v^{-\frac{5}{4}} v' + v^{-\frac{1}{4}} &= x^2 v^{-\frac{5}{4}} \\ -\frac{1}{4} v' + v &= x^2 \end{aligned}$$

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-\frac{1}{4} v^{-\frac{5}{4}} v' + \left(v^{-\frac{1}{4}} \right) &= x^2 \left(v^{-\frac{1}{4}} \right)^5 \\
-\frac{1}{4} v^{-\frac{5}{4}} v' + v^{-\frac{1}{4}} &= x^2 v^{-\frac{5}{4}} \\
-\frac{1}{4} v' + v &= x^2 \\
v' - 4v &= -4x^2
\end{aligned}$$

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$$v' - 4v = -4x^2$$

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$$\begin{aligned} v' - 4v &= -4x^2 \\ \mu(x) = e^{\int p(x)\,dx} = e^{\int -4\,dx} &= e^{-4x} \\ e^{-4x}\left(v' - 4v\right) &= -4x^2 e^{-4x} \end{aligned}$$

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\left(e^{-4x}v\right)' &= -4x^2 e^{-4x} \\
e^{-4x}v &= \int -4x^2 e^{-4x}\,dx \\
v &= -4e^{4x}\int x^2 e^{-4x}\,dx
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$$\int x^2 e^{-4x}\,dx = -\frac{1}{4}e^{-4x}x^2 - \int \left(-\frac{1}{4}\right) e^{-4x} 2x\,dx$$

Solve the Initial Value Problem $y' + y = x^2 y^5$, $y(0) = 1$.

Solving the linear equation (continued).

$$\begin{aligned} \int x^2 e^{-4x}\,dx &= -\frac{1}{4}e^{-4x}x^2 - \int \left(-\frac{1}{4}\right) e^{-4x} 2x\,dx \\ &= -\frac{1}{4}e^{-4x}x^2 + \frac{1}{2}\int e^{-4x} x\,dx \end{aligned}$$

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&= -\frac{1}{4}e^{-4x}x^2 + \frac{1}{2}\int e^{-4x}x\,dx \\
&= -\frac{1}{4}e^{-4x}x^2 + \frac{1}{2}\left[-\frac{1}{4}e^{-4x}x - \int -\frac{1}{4}e^{-4x}\,dx\right]
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&= -\frac{1}{4}e^{-4x}x^2 + \frac{1}{2}\left[-\frac{1}{4}e^{-4x}x - \int -\frac{1}{4}e^{-4x}\,dx\right] \\
&= -\frac{1}{4}e^{-4x}x^2 - \frac{1}{8}e^{-4x}x + \frac{1}{8}\int e^{-4x}\,dx
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Solving the linear equation (concluded), general solution of the original equation.

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$$v = -4e^{4x}\left[-\frac{1}{4}e^{-4x}x^2 - \frac{1}{8}e^{-4x}x - \frac{1}{32}e^{-4x} + c\right]$$

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$$y = v^{-\frac{1}{4}}$$

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\begin{aligned}
v &= -4e^{4x}\left[-\frac{1}{4}e^{-4x}x^2 - \frac{1}{8}e^{-4x}x - \frac{1}{32}e^{-4x} + c\right] \\
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\end{aligned}
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y = v^{-\frac{1}{4}} = \left(x^2 + \frac{1}{2}x + \frac{1}{8} + ce^{4x}\right)^{-\frac{1}{4}}
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Finding c.

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$$1 = y(0)$$

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Finding c.

$$\begin{aligned} y &= \left(x^2 + \frac{1}{2}x + \frac{1}{8} + ce^{4x}\right)^{-\frac{1}{4}} \\ 1 &= y(0) = \left(0^2 + \frac{1}{2}\cdot 0 + \frac{1}{8} + ce^{4\cdot 0}\right)^{-\frac{1}{4}} \end{aligned}$$

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\begin{aligned}
y &= \left(x^2 + \frac{1}{2}x + \frac{1}{8} + ce^{4x} \right)^{-\frac{1}{4}} \\
1 &= y(0) = \left(0^2 + \frac{1}{2} \cdot 0 + \frac{1}{8} + ce^{4 \cdot 0} \right)^{-\frac{1}{4}} \\
1 &= \left(\frac{1}{8} + c \right)^{-\frac{1}{4}}
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\end{aligned}
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1 &= \left(\frac{1}{8} + c\right)^{-\frac{1}{4}} \\
1 &= \frac{1}{8} + c \\
c &= \frac{7}{8}, \qquad y = \left(x^2 + \frac{1}{2}x + \frac{1}{8} + \frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}
\end{aligned}$$

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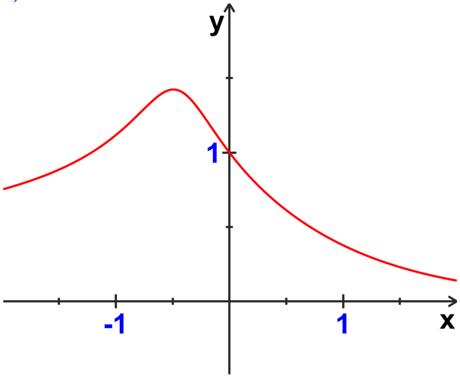

$$y = \left(x^2 + \frac{1}{2}x + \frac{1}{8} + \frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$$

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&\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
&= -\frac{1}{4}\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(2x+\frac{1}{2}+\frac{7}{2}e^{4x}\right)+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}
\end{aligned}
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Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$\begin{aligned}
&\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
&= -\frac{1}{4}\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(2x+\frac{1}{2}+\frac{7}{2}e^{4x}\right)+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
&= \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(-\frac{x}{2}-\frac{1}{8}-\frac{7}{8}e^{4x}+x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)
\end{aligned}$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$$

$$= -\frac{1}{4}\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(2x+\frac{1}{2}+\frac{7}{2}e^{4x}\right)+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$$

$$= \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(-\frac{x}{2}-\frac{1}{8}-\frac{7}{8}e^{4x}+x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)$$

$$= \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}x^2$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$\begin{aligned}
&\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
&= -\frac{1}{4}\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(2x+\frac{1}{2}+\frac{7}{2}e^{4x}\right)+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
&= \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(-\frac{x}{2}-\frac{1}{8}-\frac{7}{8}e^{4x}+x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)\\
&= \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}x^2 \quad = \quad y^5x^2
\end{aligned}$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5,\ y(0)=1$?

$$
\begin{aligned}
&\frac{d}{dx}\left[\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\right]+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
= &\ -\frac{1}{4}\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(2x+\frac{1}{2}+\frac{7}{2}e^{4x}\right)+\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}\\
= &\ \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}\left(-\frac{x}{2}-\frac{1}{8}-\frac{7}{8}e^{4x}+x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)\\
= &\ \left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{5}{4}}x^2 \quad = \quad y^5x^2 \qquad \surd
\end{aligned}
$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$y(0) = \left(0^2+\frac{1}{2}\cdot 0+\frac{1}{8}+\frac{7}{8}e^{4\cdot 0}\right)^{-\frac{1}{4}}$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5$, $y(0)=1$?

$$\begin{aligned} y(0) &= \left(0^2+\frac{1}{2}\cdot 0+\frac{1}{8}+\frac{7}{8}e^{4\cdot 0}\right)^{-\frac{1}{4}} \\ &= \left(\frac{1}{8}+\frac{7}{8}\right)^{-\frac{1}{4}} \end{aligned}$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5,\ y(0)=1$?

$$\begin{aligned} y(0) &= \left(0^2+\frac{1}{2}\cdot 0+\frac{1}{8}+\frac{7}{8}e^{4\cdot 0}\right)^{-\frac{1}{4}} \\ &= \left(\frac{1}{8}+\frac{7}{8}\right)^{-\frac{1}{4}} \\ &= 1 \end{aligned}$$

Does $y = \left(x^2 + \frac{1}{2}x + \frac{1}{8} + \frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y' + y = x^2y^5$, $y(0) = 1$?

$$
\begin{aligned}
y(0) &= \left(0^2 + \frac{1}{2}\cdot 0 + \frac{1}{8} + \frac{7}{8}e^{4\cdot 0}\right)^{-\frac{1}{4}} \\
&= \left(\frac{1}{8} + \frac{7}{8}\right)^{-\frac{1}{4}} \\
&= 1 \qquad \surd
\end{aligned}
$$

Does $y=\left(x^2+\frac{1}{2}x+\frac{1}{8}+\frac{7}{8}e^{4x}\right)^{-\frac{1}{4}}$ Really Solve the Initial Value Problem $y'+y=x^2y^5,\ y(0)=1$?

$$
\begin{aligned}
y(0) &= \left(0^2+\frac{1}{2}\cdot 0+\frac{1}{8}+\frac{7}{8}e^{4\cdot 0}\right)^{-\frac{1}{4}}\\
&= \left(\frac{1}{8}+\frac{7}{8}\right)^{-\frac{1}{4}}\\
&= 1 \qquad \surd
\end{aligned}
$$

Yes, it does.