Euler's Method

Bernd Schröder

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3. At the end point (x_1, y_1) of this line segment we use $F(x_1, y_1)$ to compute the slope at this new point (x_1, y_1) and then we follow this new line segment.
4. This process repeats for as far as we want to go.

Euler's Method from a Direction Field

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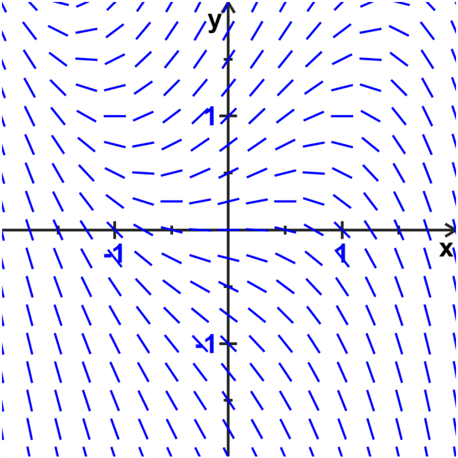

$$y' = y - x^2, \; y(0) = 0$$

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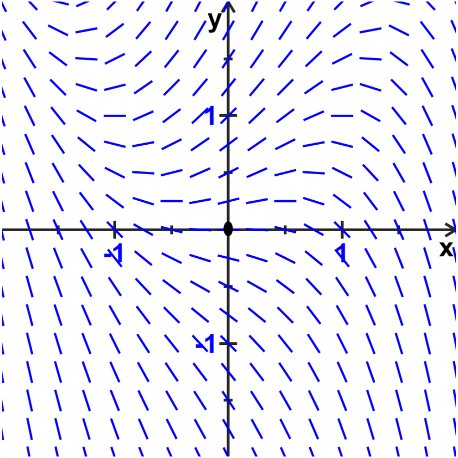

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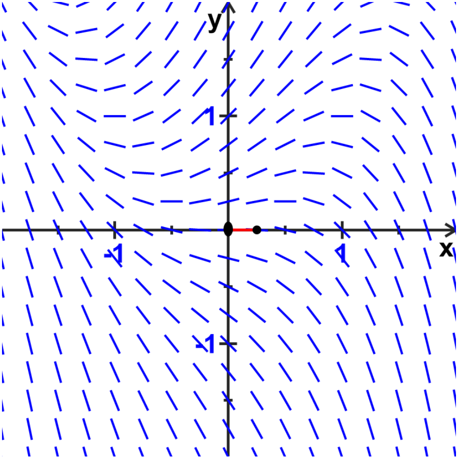

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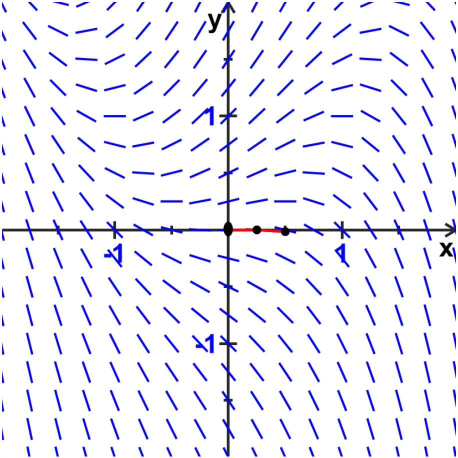

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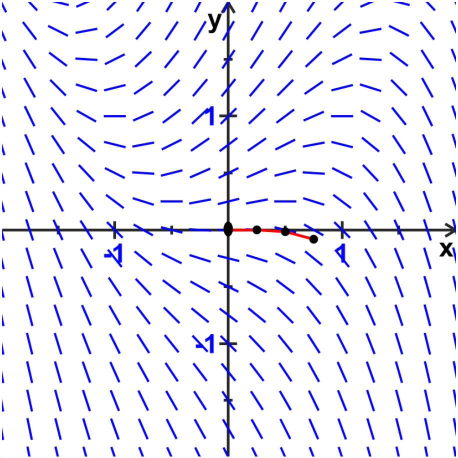

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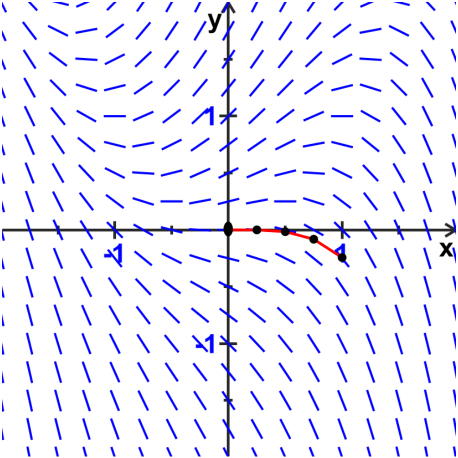

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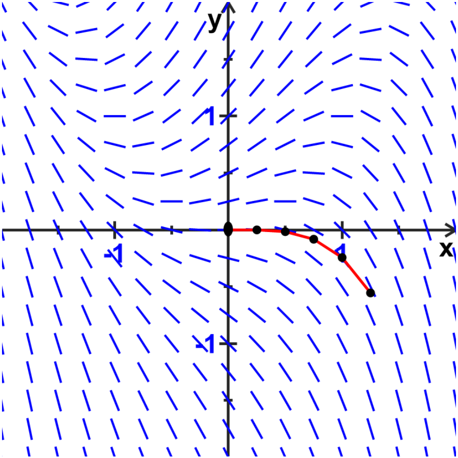

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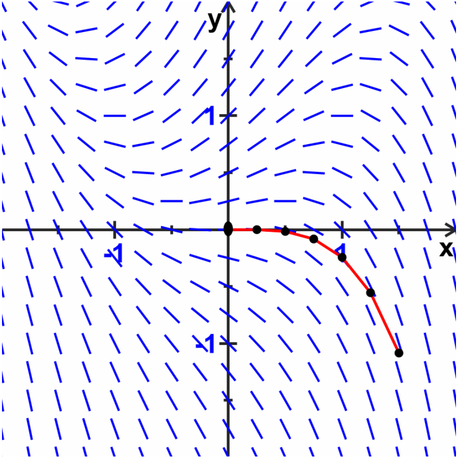

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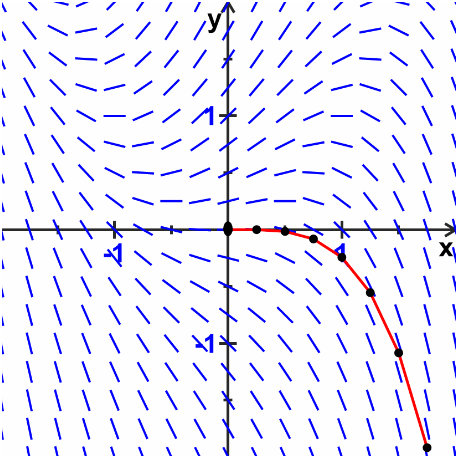

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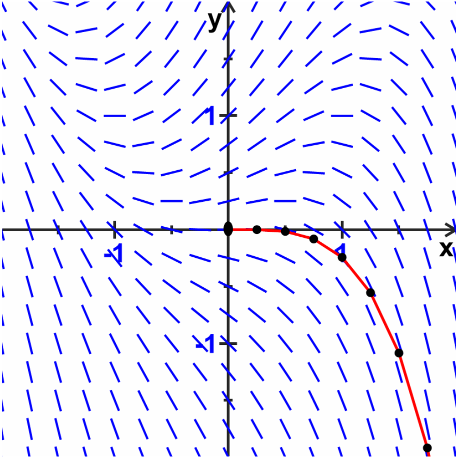

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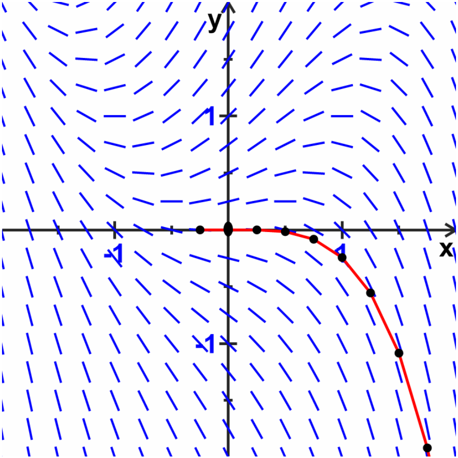

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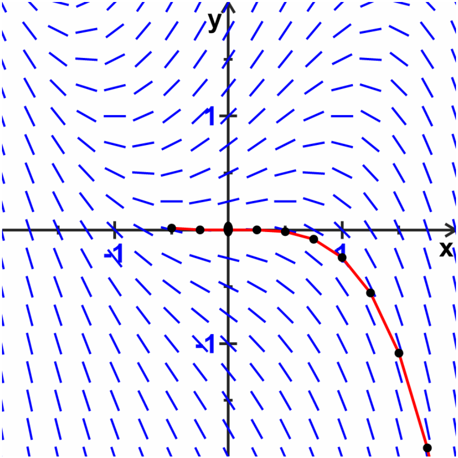

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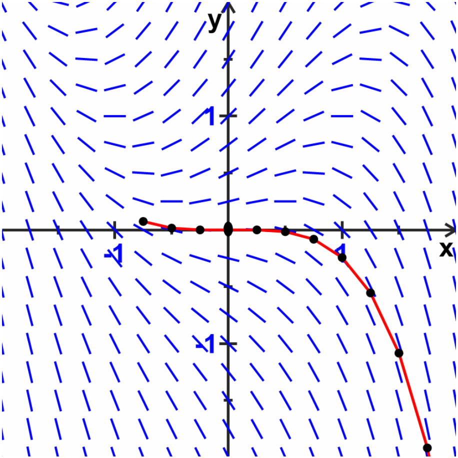

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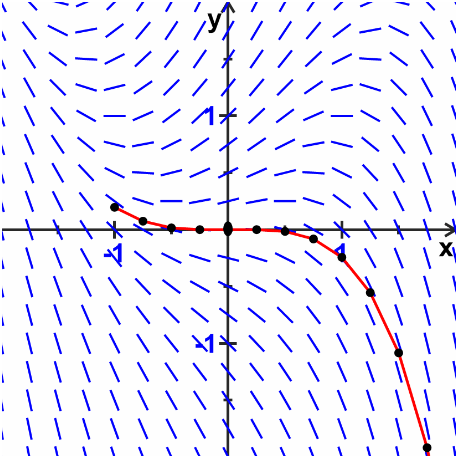

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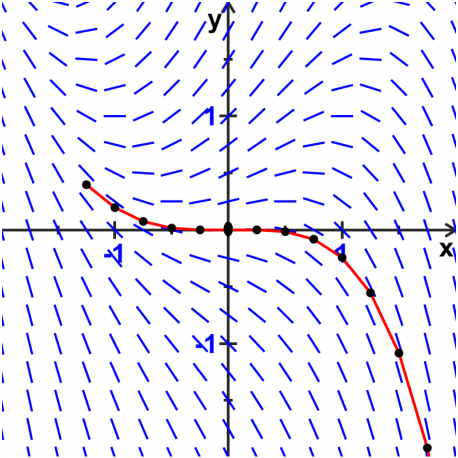

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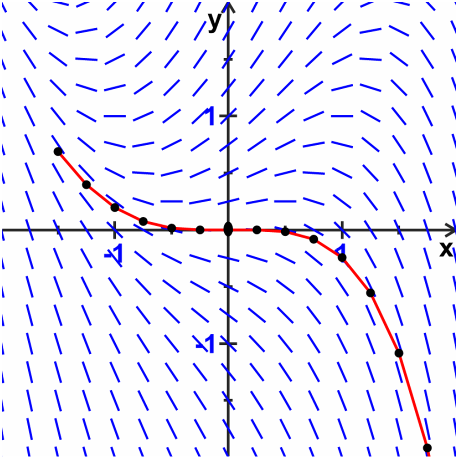

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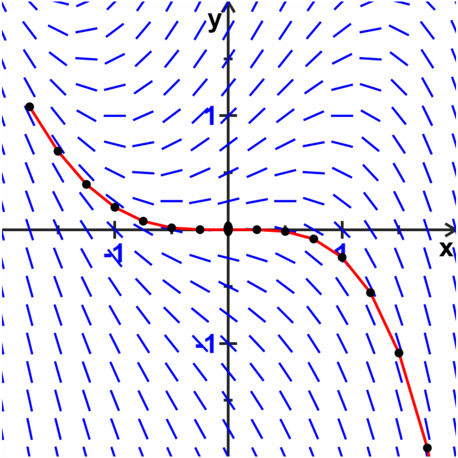

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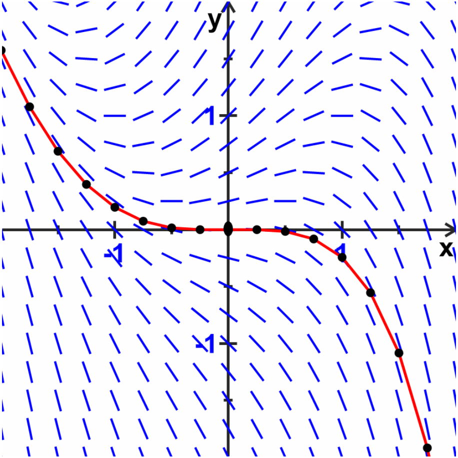

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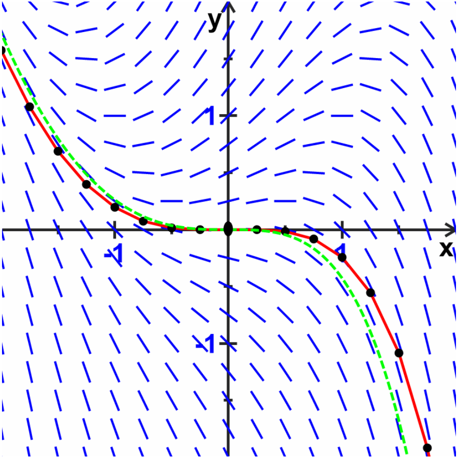

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400 pts.

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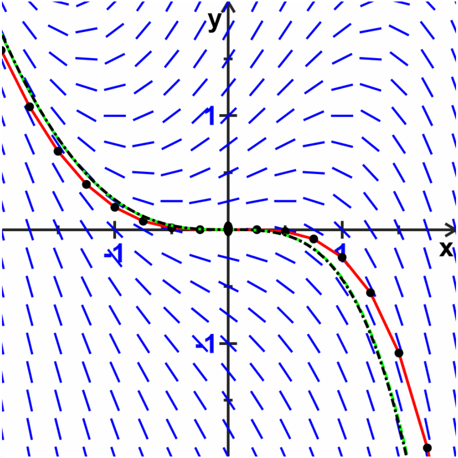

$y' = y - x^2,\ y(0) = 0,$
400 pts., exact solution: $y(x) = x^2 + 2x + 2 - 2e^x$

Reminder for Spreadsheet Implementation

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Step length: Δx. Initial values: (x_0, y_0).

$$\begin{aligned} x_{n+1} &:= x_n + \Delta x \\ y_{n+1} &= y_n + F(x_n, y_n)\Delta x. \end{aligned}$$

The value y_n will be an approximation for the value of the solution y at x_n.