Laplace Transforms of Damped Trigonometric Functions

Bernd Schröder

Everything Remains As It Was

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No matter what functions arise, the idea for solving differential equations with Laplace transforms stays the same.

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Time Domain (t)

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Time Domain (t)

Original DE & IVP

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Time Domain (t)

Original DE & IVP	$\xrightarrow{\mathscr{L}}$	Algebraic equation for the Laplace transform

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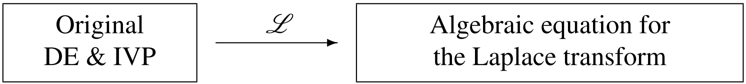
Time Domain (t) Transform domain (s)



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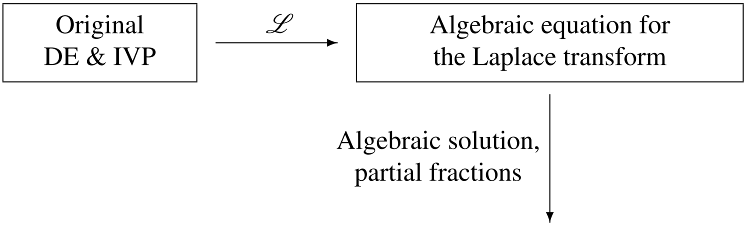

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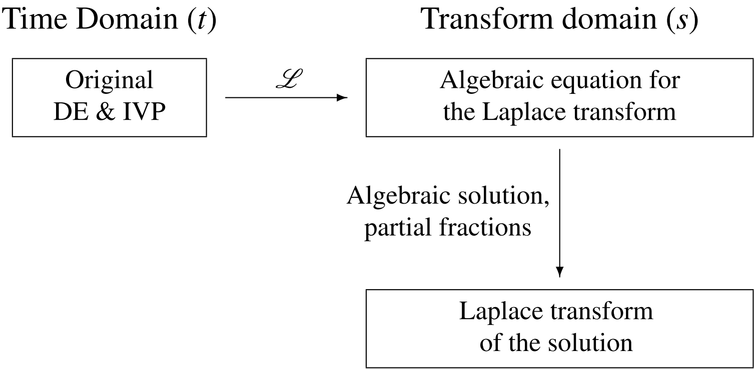

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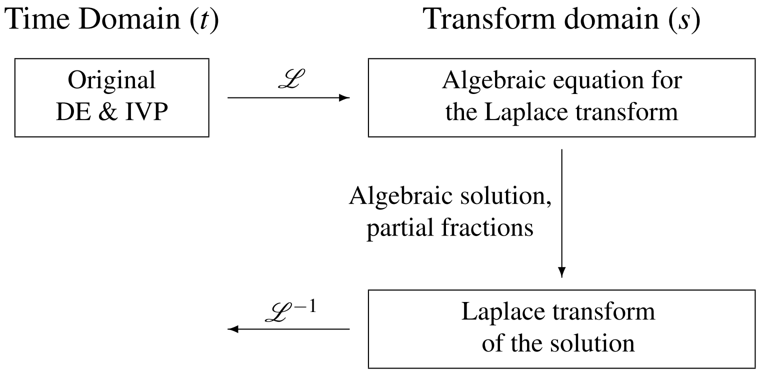

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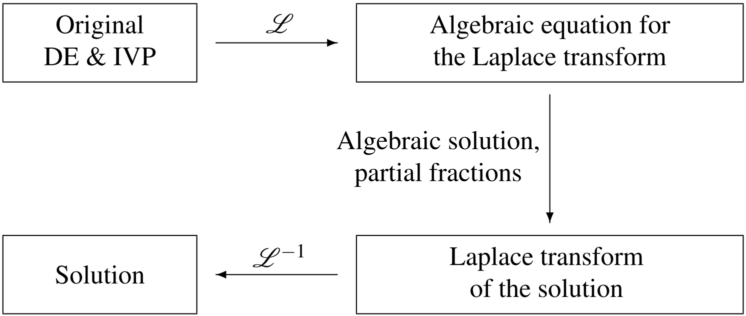

Solve the Initial Value Problem

$y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

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Finding the Laplace transform of the solution.

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Finding the Laplace transform of the solution.

$$\begin{aligned} y'' + 2y' + 2y &= \sin(2t),\ y(0) = 1,\ y'(0) = 0 \\ s^2Y - s + 2sY - 2 + 2Y &= \frac{2}{s^2+4} \end{aligned}$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t),\ y(0)=1,\ y'(0)=0$

Finding the Laplace transform of the solution.

$$\begin{aligned} y''+2y'+2y &= \sin(2t),\ y(0)=1,\ y'(0)=0 \\ s^2Y-s+2sY-2+2Y &= \frac{2}{s^2+4} \\ \left(s^2+2s+2\right)Y &= s+2+\frac{2}{s^2+4} \end{aligned}$$

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\left(s^2+2s+2\right)Y &= \frac{(s+2)\left(s^2+4\right)+2}{s^2+4}
\end{aligned}$$

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\left(s^2+2s+2\right)Y &= \frac{(s+2)\left(s^2+4\right)+2}{s^2+4}=\frac{s^3+2s^2+4s+10}{s^2+4}
\end{aligned}$$

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\left(s^2+2s+2\right)Y &= \frac{(s+2)\left(s^2+4\right)+2}{s^2+4}=\frac{s^3+2s^2+4s+10}{s^2+4}\\
Y &= \frac{s^3+2s^2+4s+10}{\left(s^2+2s+2\right)\left(s^2+4\right)}
\end{aligned}$$

Solve the Initial Value Problem

$$y''+2y'+2y=\sin(2t),\ y(0)=1,\ y'(0)=0$$

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$y'' + 2y' + 2y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$

Partial fraction decomposition.

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$$Y=\frac{s^3+2s^2+4s+10}{\left(s^2+2s+2\right)\left(s^2+4\right)} \quad =$$

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$$Y = \frac{s^3 + 2s^2 + 4s + 10}{(s^2 + 2s + 2)(s^2 + 4)} = \frac{As + B}{s^2 + 2s + 2} + \frac{Cs + D}{s^2 + 4}$$

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$$s^3 + 2s^2 + 4s + 10 = (As+B)\left(s^2+4\right) + (Cs+D)\left(s^2+2s+2\right)$$

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s^3+2s^2+4s+10 &= (A+C)s^3+(B+2C+D)s^2\\
&\quad +(4A+2C+2D)s
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s^3 + 2s^2 + 4s + 10 &= (A + C)s^3 + (B + 2C + D)s^2 \\
&\quad + (4A + 2C + 2D)s + (4B + 2D) \\
A + C &= 1 \\
B + 2C + D &= 2
\end{aligned}
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$$D=-\frac{1}{5},$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

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$$D=-\frac{1}{5},\ C=-\frac{1}{5},$$

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$$D = -\frac{1}{5},\ C = -\frac{1}{5},\ B = \frac{13}{5},$$

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$$D=-\frac{1}{5},\ C=-\frac{1}{5},\ B=\frac{13}{5},\ A=\frac{6}{5},$$

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$$D = -\frac{1}{5}, C = -\frac{1}{5}, B = \frac{13}{5}, A = \frac{6}{5},$$

$$Y = \frac{\frac{6}{5}s + \frac{13}{5}}{s^2 + 2s + 2} + \frac{-\frac{1}{5}s - \frac{1}{5}}{s^2 + 4}$$

Solve the Initial Value Problem

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Inverting the Laplace transform.

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

Inverting the Laplace transform.

$$Y = \frac{\frac{6}{5}s+\frac{13}{5}}{s^2+2s+2}+\frac{-\frac{1}{5}s-\frac{1}{5}}{s^2+4}$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

Inverting the Laplace transform.

$$Y = \frac{\frac{6}{5}s+\frac{13}{5}}{s^2+2s+2}+\frac{-\frac{1}{5}s-\frac{1}{5}}{s^2+4}=\frac{1}{5}\frac{6s+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{5}\frac{1}{s^2+4}$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

Inverting the Laplace transform.

$$\begin{aligned} Y &= \frac{\frac{6}{5}s+\frac{13}{5}}{s^2+2s+2}+\frac{-\frac{1}{5}s-\frac{1}{5}}{s^2+4}=\frac{1}{5}\frac{6s+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{5}\frac{1}{s^2+4} \\ &= \frac{1}{5}\frac{6(s+1-1)+13}{(s+1)^2+1} \end{aligned}$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

Inverting the Laplace transform.

$$
\begin{aligned}
Y &= \frac{\frac{6}{5}s+\frac{13}{5}}{s^2+2s+2}+\frac{-\frac{1}{5}s-\frac{1}{5}}{s^2+4}=\frac{1}{5}\frac{6s+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{5}\frac{1}{s^2+4} \\
&= \frac{1}{5}\frac{6(s+1-1)+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}
\end{aligned}
$$

Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

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Solve the Initial Value Problem $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

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Solve the Initial Value Problem
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&= \frac{1}{5}\frac{6(s+1)-6+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{10}\frac{2}{s^2+4}
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 2y' + 2y = \sin(2t),\ y(0) = 1,\ y'(0) = 0$

Inverting the Laplace transform.

$$
\begin{aligned}
Y &= \frac{\frac{6}{5}s + \frac{13}{5}}{s^2+2s+2} + \frac{-\frac{1}{5}s - \frac{1}{5}}{s^2+4} = \frac{1}{5}\frac{6s+13}{(s+1)^2+1} - \frac{1}{5}\frac{s}{s^2+4} - \frac{1}{5}\frac{1}{s^2+4} \\
&= \frac{1}{5}\frac{6((s+1)-1)+13}{(s+1)^2+1} - \frac{1}{5}\frac{s}{s^2+4} - \frac{1}{5}\frac{2\cdot\frac{1}{2}}{s^2+4} \\
&= \frac{1}{5}\frac{6(s+1)-6+13}{(s+1)^2+1} - \frac{1}{5}\frac{s}{s^2+4} - \frac{1}{10}\frac{2}{s^2+4} \\
&= \frac{6}{5}\frac{s+1}{(s+1)^2+1} + \frac{7}{5}\frac{1}{(s+1)^2+1} - \frac{1}{5}\frac{s}{s^2+4} - \frac{1}{10}\frac{2}{s^2+4}
\end{aligned}
$$

Solve the Initial Value Problem

$y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$

Inverting the Laplace transform.

$$\begin{aligned}
Y &= \frac{\frac{6}{5}s+\frac{13}{5}}{s^2+2s+2}+\frac{-\frac{1}{5}s-\frac{1}{5}}{s^2+4} = \frac{1}{5}\frac{6s+13}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{5}\frac{1}{s^2+4}\\
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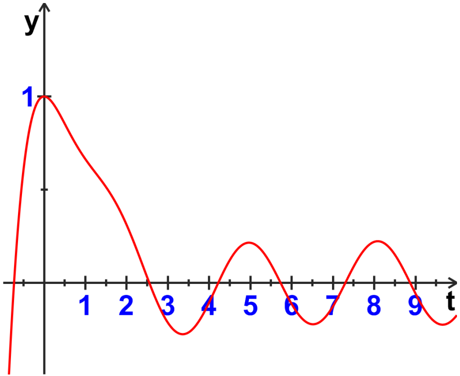
&= \frac{6}{5}\frac{s+1}{(s+1)^2+1}+\frac{7}{5}\frac{1}{(s+1)^2+1}-\frac{1}{5}\frac{s}{s^2+4}-\frac{1}{10}\frac{2}{s^2+4}\\
y &= \frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)
\end{aligned}$$

Solve the Initial Value Problem

$$y'' + 2y' + 2y = \sin(2t),\ y(0) = 1,\ y'(0) = 0$$



$$y = \frac{6}{5}e^{-t}\cos(t) + \frac{7}{5}e^{-t}\sin(t) - \frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t)$$

Does $y = \frac{6}{5}e^{-t}\cos(t) + \frac{7}{5}e^{-t}\sin(t) - \frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t)$ Solve $y'' + 2y' + 2y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$?

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$2y+2y'+y''$

Does $y=\frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)$ Solve $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$?

$$2y+2y'+y'' \quad = \quad 2\left(\frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)\right)$$

Does $y=\frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)$ Solve $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}2y+2y'+y'' &= 2\left(\frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)\right)\\ &\quad +2\left(\frac{1}{5}e^{-t}\cos(t)-\frac{13}{5}e^{-t}\sin(t)+\frac{2}{5}\sin(2t)-\frac{2}{10}\cos(2t)\right)\end{aligned}$$

Does $y = \frac{6}{5}e^{-t}\cos(t) + \frac{7}{5}e^{-t}\sin(t) - \frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t)$ Solve $y'' + 2y' + 2y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$?

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$$
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$$

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&= \sin(2t) \qquad \surd \\
y(0) &= 1 \qquad \surd \\
y'(0) &= 0 \qquad \surd
\end{aligned}$$

or use a computer algebra system.

Does $y = \frac{6}{5}e^{-t}\cos(t) + \frac{7}{5}e^{-t}\sin(t) - \frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t)$ Solve $y'' + 2y' + 2y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$?

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$$y(t) := \frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)$$

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$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + 2\,y(t) \text{ simplify } \rightarrow \sin(2\cdot t)$$

Does $y = \frac{6}{5}e^{-t}\cos(t) + \frac{7}{5}e^{-t}\sin(t) - \frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t)$ Solve $y'' + 2y' + 2y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$?

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$$y(0) \text{ simplify } \rightarrow 1$$

$$yp(t) := \frac{d}{dt}y(t)$$

Does $y=\frac{6}{5}e^{-t}\cos(t)+\frac{7}{5}e^{-t}\sin(t)-\frac{1}{5}\cos(2t)-\frac{1}{10}\sin(2t)$ Solve $y''+2y'+2y=\sin(2t)$, $y(0)=1$, $y'(0)=0$?

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$$\frac{d^2}{dt^2}y(t)+2\frac{d}{dt}y(t)+2\,y(t) \text{ simplify } \rightarrow \sin(2\cdot t)$$

$$y(0) \text{ simplify } \rightarrow 1$$

$$yp(t) := \frac{d}{dt}y(t)$$

$$yp(0) \text{ simplify } \rightarrow 0$$