The Laplace Transform of The Dirac Delta Function

Bernd Schröder

Everything Remains As It Was

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Original DE & IVP

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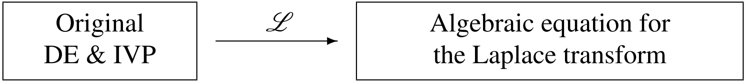

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Time Domain (t) | Transform domain (s)

Original DE & IVP $\xrightarrow{\mathscr{L}}$ Algebraic equation for the Laplace transform

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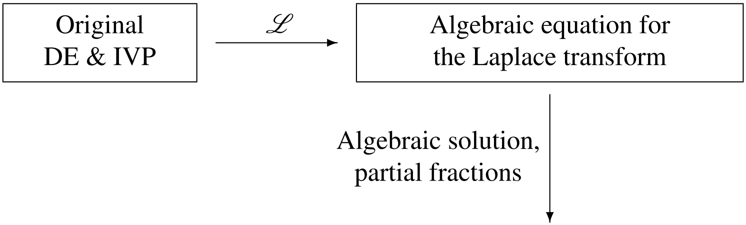

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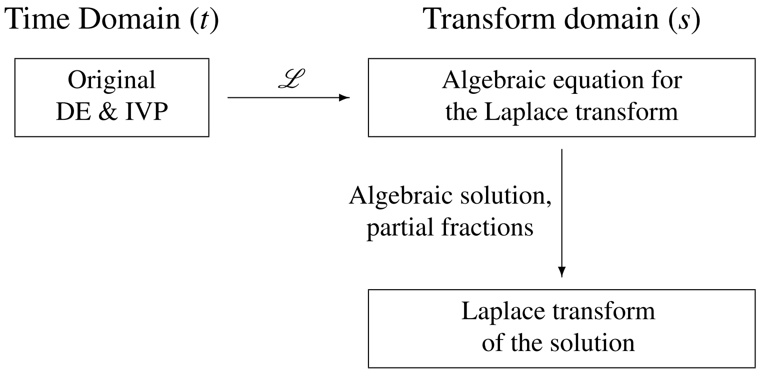

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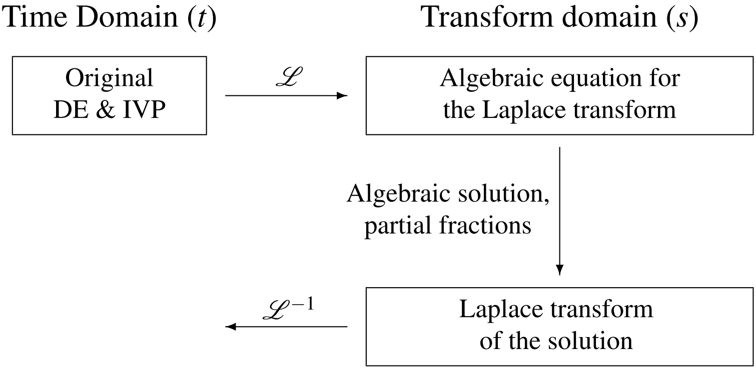

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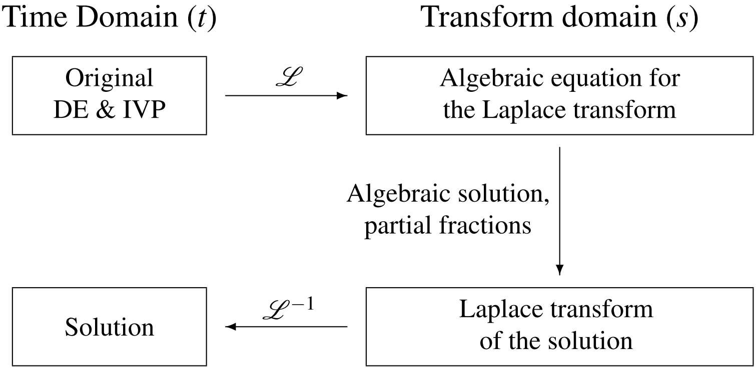

What is the Delta Function?

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4. $\mathscr{L}\left\{\delta(t-a)\right\} = e^{-as}$

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In an LRC circuit with $L = 1H$, $R = 8\Omega$ and $C = \frac{1}{15}F$, the capacitor initially carries a charge of $1C$ and no currents are flowing. There is no external voltage source. At time $t = 2s$, a power surge instantaneously applies an impulse of $4\delta(t-2)$ into the system.

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Solve the Initial Value Problem

$q'' + 8q' + 15q = 4\delta(t-2),\ q(0) = 1,\ q'(0) = 0$

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$$s^2Q-s$$

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$$\begin{aligned} q''+8q'+15q &= 4\delta(t-2) \\ s^2Q-s+8sQ-8 & \end{aligned}$$

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$$\begin{aligned} q'' + 8q' + 15q &= 4\delta(t-2) \\ s^2Q - s + 8sQ - 8 + 15Q &= 4e^{-2s} \\ \left(s^2 + 8s + 15\right)Q &= s + 8 + 4e^{-2s} \end{aligned}$$

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\left(s^2+8s+15\right)Q &= s+8+4e^{-2s}\\
&\quad \text{(keep the exponential separate)}\\
Q &= \frac{s+8}{s^2+8s+15}+e^{-2s}\frac{4}{s^2+8s+15}
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& \quad \text{(keep the exponential separate)} \\
Q &= \frac{s+8}{s^2+8s+15} + e^{-2s}\frac{4}{s^2+8s+15} \\
Q &= \frac{s+8}{(s+3)(s+5)} + e^{-2s}\frac{4}{(s+3)(s+5)}
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Partial fraction decompositions.

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s=-5 &\qquad 3=-2B, \qquad B=-\frac{3}{2}\\
\frac{s+8}{(s+3)(s+5)} &= \frac{5}{2}\frac{1}{s+3}-\frac{3}{2}\frac{1}{s+5}
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q &= \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-5t} + \mathscr{U}(t-2)
\end{aligned}
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&= \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-5t} + \mathscr{U}(t-2)\left[2e^{-3(t-2)} - 2e^{-5(t-2)}\right]
\end{aligned}
$$

What Happens in the Physical System?

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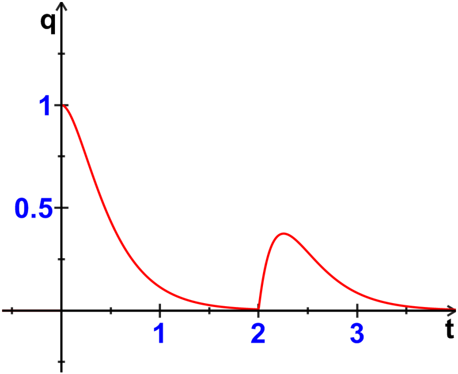

$$q=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}+\mathscr{U}(t-2)\left[2e^{-3(t-2)}-2e^{-5(t-2)}\right]$$

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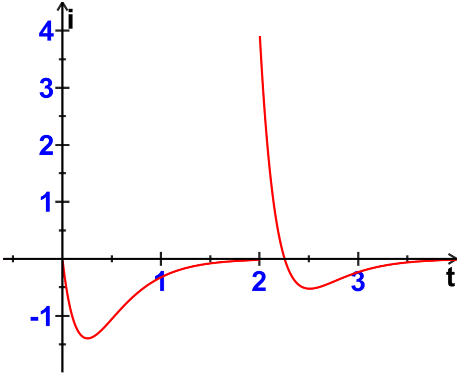

$$q' = -\frac{15}{2}e^{-3t} + \frac{15}{2}e^{-5t} + \mathscr{U}(t-2)\left[-6e^{-3(t-2)} + 10e^{-5(t-2)}\right]$$

Does $q=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}+\mathscr{U}(t-2)\left[2e^{-3(t-2)}-2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q''+8q'+15q=4\delta(t-2)$, $q(0)=1$, $q'(0)=0$?

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First consider $q_1 = \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-5t}$.

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$$=\quad\left(\frac{75}{2}-\frac{120}{2}+\frac{45}{2}\right)e^{-3t}+\left(-\frac{45}{2}+\frac{120}{2}-\frac{75}{2}\right)e^{-5t}$$

Does $q=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}+\mathscr{U}(t-2)\left[2e^{-3(t-2)}-2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q''+8q'+15q=4\delta(t-2)$, $q(0)=1$, $q'(0)=0$?

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&\qquad = \left(\frac{75}{2}-\frac{120}{2}+\frac{45}{2}\right)e^{-3t}+\left(-\frac{45}{2}+\frac{120}{2}-\frac{75}{2}\right)e^{-5t}\\
&\qquad = 0 \qquad \surd
\end{aligned}$$

Does $q = \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-5t} + \mathscr{U}(t-2)\left[2e^{-3(t-2)} - 2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q'' + 8q' + 15q = 4\delta(t-2)$, $q(0) = 1$, $q'(0) = 0$?

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$$= \left(\frac{75}{2} - \frac{120}{2} + \frac{45}{2}\right)e^{-3t} + \left(-\frac{45}{2} + \frac{120}{2} - \frac{75}{2}\right)e^{-5t}$$

$$= 0 \qquad \surd$$

$q_1(0)$

Does $q=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}+\mathscr{U}(t-2)\left[2e^{-3(t-2)}-2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q''+8q'+15q=4\delta(t-2)$, $q(0)=1$, $q'(0)=0$?

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$$= \left(\frac{75}{2}-\frac{120}{2}+\frac{45}{2}\right)e^{-3t}+\left(-\frac{45}{2}+\frac{120}{2}-\frac{75}{2}\right)e^{-5t}$$

$$= 0 \qquad \surd$$

$$q_1(0) = \frac{5}{2}e^{-3\cdot 0}-\frac{3}{2}e^{-5\cdot 0}$$

Does $q = \frac{5}{2}e^{-3t} - \frac{3}{2}e^{-5t} + \mathscr{U}(t-2)\left[2e^{-3(t-2)} - 2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q'' + 8q' + 15q = 4\delta(t-2)$, $q(0) = 1$, $q'(0) = 0$?

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$$= \left(\frac{75}{2} - \frac{120}{2} + \frac{45}{2}\right)e^{-3t} + \left(-\frac{45}{2} + \frac{120}{2} - \frac{75}{2}\right)e^{-5t}$$
$$= 0 \qquad \surd$$

$$q_1(0) = \frac{5}{2}e^{-3\cdot 0} - \frac{3}{2}e^{-5\cdot 0} = 1$$

Does $q=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}+\mathscr{U}(t-2)\left[2e^{-3(t-2)}-2e^{-5(t-2)}\right]$ Solve the Initial Value Problem $q''+8q'+15q=4\delta(t-2)$, $q(0)=1$, $q'(0)=0$?

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$$= \left(\frac{75}{2}-\frac{120}{2}+\frac{45}{2}\right)e^{-3t}+\left(-\frac{45}{2}+\frac{120}{2}-\frac{75}{2}\right)e^{-5t}$$

$$= 0 \qquad \surd$$

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$$
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\end{aligned}
$$

$$
q_1(0) = \frac{5}{2}e^{-3\cdot 0}-\frac{3}{2}e^{-5\cdot 0}=1 \qquad \surd
$$

$$
q_1'(0)
$$

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&= \left(\frac{75}{2}-\frac{120}{2}+\frac{45}{2}\right)e^{-3t}+\left(-\frac{45}{2}+\frac{120}{2}-\frac{75}{2}\right)e^{-5t}\\
&= 0 \qquad \surd
\end{aligned}
$$

$$
\begin{aligned}
q_1(0) &= \frac{5}{2}e^{-3\cdot 0}-\frac{3}{2}e^{-5\cdot 0}=1 \qquad \surd\\
q_1'(0) &= -\frac{15}{2}e^{-3\cdot 0}+\frac{15}{2}e^{-5\cdot 0}
\end{aligned}
$$

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$$15\left(2e^{-3(t-2)}-2e^{-5(t-2)}\right)+8\left(-6e^{-3(t-2)}+10e^{-5(t-2)}\right)+\left(18e^{-3(t-2)}-50e^{-5(t-2)}\right)$$

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&= (30-48+18)e^{-3(t-2)}+(-30+80-50)e^{-5(t-2)}\\
&= 0 \qquad \surd
\end{aligned}$$