Laplace Transforms of Periodic Functions

Bernd Schröder

Everything Remains As It Was

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Original DE & IVP

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Original DE & IVP $\xrightarrow{\mathscr{L}}$

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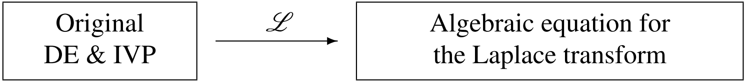
Time Domain (t)



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Time Domain (t) | Transform domain (s)

Original DE & IVP $\xrightarrow{\mathscr{L}}$ Algebraic equation for the Laplace transform

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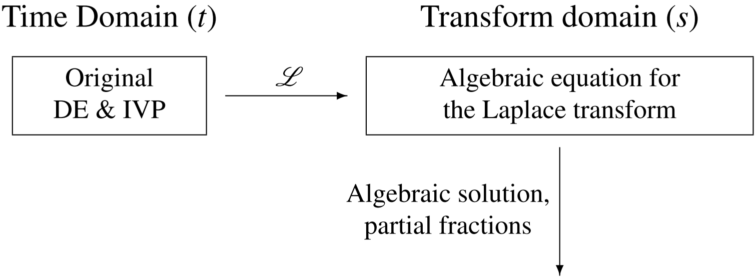

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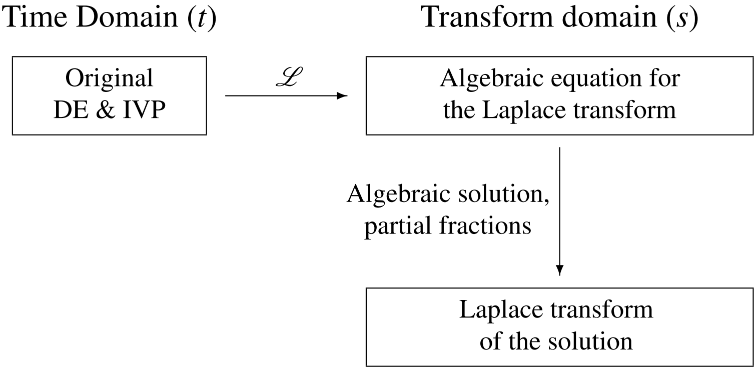

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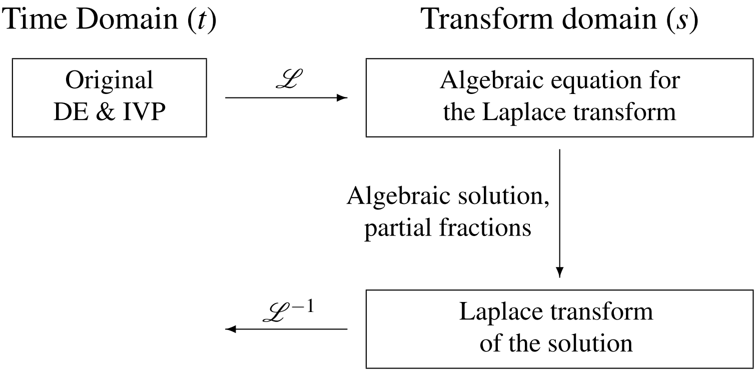

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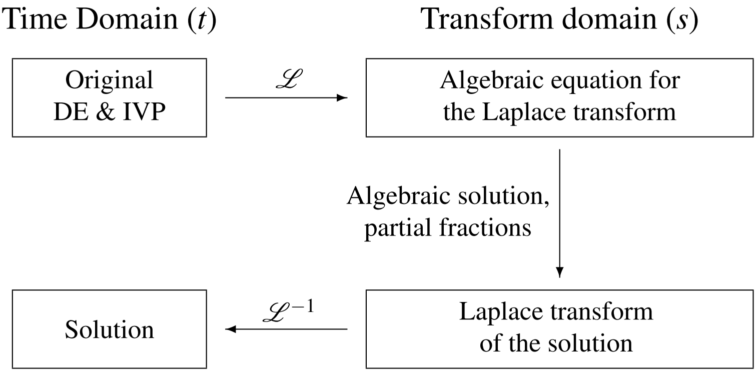

Periodic Functions

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1. A function f is periodic with period $T > 0$ if and only if for all t we have $f(t+T) = f(t)$.
2. If f is bounded, piecewise continuous and periodic with period T, then

$$\mathscr{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

How Did We Get That?

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$$\mathscr{L}\left\{f(t)\right\}$$

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$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

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$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} e^{-st} f(t)\, dt$$

How Did We Get That?

$$\begin{aligned}\mathscr{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-st}f(t)\,dt \\ &= \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-s\big((t-nT)+nT\big)}f(t)\,dt\end{aligned}$$

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$$\begin{aligned}\mathscr{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-st}f(t)\,dt \\ &= \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-s\big((t-nT)+nT\big)}f(t)\,dt = \sum_{n=0}^{\infty}\int_0^T e^{-s(u+nT)}f(u)\,du\end{aligned}$$

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$$\begin{aligned}\mathcal{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-st}f(t)\,dt \\ &= \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-s\left((t-nT)+nT\right)}f(t)\,dt = \sum_{n=0}^{\infty}\int_0^T e^{-s(u+nT)}f(u)\,du \\ &= \sum_{n=0}^{\infty} e^{-nsT}\int_0^T e^{-su}f(u)\,du\end{aligned}$$

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$$\begin{aligned}\mathscr{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-st}f(t)\,dt \\ &= \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-s\big((t-nT)+nT\big)}f(t)\,dt = \sum_{n=0}^{\infty}\int_0^T e^{-s(u+nT)}f(u)\,du \\ &= \sum_{n=0}^{\infty} e^{-nsT}\int_0^T e^{-su}f(u)\,du = \left[\sum_{n=0}^{\infty}\left(e^{-sT}\right)^n\right]\int_0^T e^{-st}f(t)\,dt\end{aligned}$$

How Did We Get That?

$$
\begin{aligned}
\mathscr{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} e^{-st}f(t)\,dt \\
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&= \sum_{n=0}^{\infty} e^{-nsT}\int_0^T e^{-su}f(u)\,du = \left[\sum_{n=0}^{\infty}\left(e^{-sT}\right)^n\right]\int_0^T e^{-st}f(t)\,dt \\
&= \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt
\end{aligned}
$$

Solve the Initial Value Problem

$$3y' + 2y = \big|\sin(t)\big|,\ y(0) = 0$$

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$$\mathscr{L}\{|\sin(t)|\} = \frac{1}{1-e^{-s\pi}} \int_0^{\pi} e^{-st} \left|\sin(t)\right| \, dt$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$\begin{aligned}\mathscr{L}\left\{\left|\sin(t)\right|\right\} &= \frac{1}{1-e^{-s\pi}}\int_0^\pi e^{-st}\left|\sin(t)\right|\,dt \\ &= \frac{1}{1-e^{-s\pi}}\int_0^\pi e^{-st}\sin(t)\,dt\end{aligned}$$

Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|, y(0)=0$

$$\begin{aligned}
\mathscr{L}\{|\sin(t)|\} &= \frac{1}{1-e^{-s\pi}}\int_0^{\pi} e^{-st}\left|\sin(t)\right|\,dt \\
&= \frac{1}{1-e^{-s\pi}}\int_0^{\pi} e^{-st}\sin(t)\,dt \\
&= \frac{1}{1-e^{-s\pi}}\int_0^{\infty} e^{-st}\big(1-\mathscr{U}(t-\pi)\big)\sin(t)\,dt
\end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$
\begin{aligned}
\mathscr{L}\{|\sin(t)|\} &= \frac{1}{1-e^{-s\pi}} \int_0^\pi e^{-st} \left|\sin(t)\right| \, dt \\
&= \frac{1}{1-e^{-s\pi}} \int_0^\pi e^{-st} \sin(t) \, dt \\
&= \frac{1}{1-e^{-s\pi}} \int_0^\infty e^{-st} \big(1 - \mathscr{U}(t-\pi)\big) \sin(t) \, dt \\
&= \frac{1}{1-e^{-s\pi}} \Big[\mathscr{L}\{\sin(t)\}
\end{aligned}
$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

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&= \frac{1}{1-e^{-s\pi}}\left[\mathscr{L}\{\sin(t)\}+\mathscr{L}\{\mathscr{U}(t-\pi)\sin(t-\pi)\}\right]
\end{aligned}
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&= \frac{1}{1-e^{-s\pi}}\left[\mathscr{L}\{\sin(t)\}+\mathscr{L}\{\mathscr{U}(t-\pi)\sin(t-\pi)\}\right] \\
&= \frac{1}{1-e^{-s\pi}}\left[\frac{1}{s^2+1}\right.
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&= \frac{1}{1-e^{-s\pi}} \int_0^{\infty} e^{-st} \left(1 - \mathscr{U}(t-\pi)\right) \sin(t) \, dt \\
&= \frac{1}{1-e^{-s\pi}} \left[\mathscr{L}\{\sin(t)\} + \mathscr{L}\{\mathscr{U}(t-\pi)\sin(t-\pi)\}\right] \\
&= \frac{1}{1-e^{-s\pi}} \left[\frac{1}{s^2+1} + e^{-\pi s}\frac{1}{s^2+1}\right]
\end{aligned}
$$

Solve the Initial Value Problem
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&= \frac{1}{1-e^{-s\pi}} \left[\mathscr{L}\{\sin(t)\} + \mathscr{L}\{\mathscr{U}(t-\pi)\sin(t-\pi)\}\right] \\
&= \frac{1}{1-e^{-s\pi}} \left[\frac{1}{s^2+1} + e^{-\pi s}\frac{1}{s^2+1}\right] \\
&= \frac{1}{1-e^{-s\pi}} \left[1 + e^{-\pi s}\right] \frac{1}{s^2+1}
\end{aligned}
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$$3sY + 2Y$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$3sY + 2Y = \frac{1}{1 - e^{-s\pi}} \left[1 + e^{-\pi s}\right] \frac{1}{s^2 + 1}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$\begin{aligned} 3sY + 2Y &= \frac{1}{1-e^{-s\pi}}\left[1+e^{-\pi s}\right]\frac{1}{s^2+1} \\ Y &= \frac{1}{1-e^{-s\pi}}\left[1+e^{-\pi s}\right]\frac{1}{\left(s^2+1\right)\left(3s+2\right)} \end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|,\, y(0) = 0$

$$
\begin{aligned}
3sY + 2Y &= \frac{1}{1-e^{-s\pi}}\left[1+e^{-\pi s}\right]\frac{1}{s^2+1} \\
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&= \sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^n\left[1+e^{-\pi s}\right]\frac{1}{\left(s^2+1\right)\left(3s+2\right)}
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&= \sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^n\left[1+e^{-\pi s}\right]\frac{1}{\left(s^2+1\right)(3s+2)}\\
&= \left[\sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^n+\sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^{n+1}\right]\frac{1}{\left(s^2+1\right)(3s+2)}
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&= \sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^n\left[1+e^{-\pi s}\right]\frac{1}{(s^2+1)(3s+2)} \\
&= \left[\sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^n + \sum_{n=0}^{\infty}\left(e^{-\pi s}\right)^{n+1}\right]\frac{1}{(s^2+1)(3s+2)} \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right]\frac{1}{(s^2+1)(3s+2)}
\end{aligned}
$$

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$$\frac{1}{(s^2+1)(3s+2)} = \frac{As+B}{s^2+1} + \frac{C}{3s+2}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$\begin{aligned} \frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1} + \frac{C}{3s+2} \\ 1 &= (As+B)(3s+2) + C\left(s^2+1\right) \end{aligned}$$

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$$s = -\frac{2}{3}:$$

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1 &= (As+B)(3s+2) + C\left(s^2+1\right) \\
s = -\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right)
\end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$

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s = -\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right) = \frac{13}{9}C
\end{aligned}$$

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\end{aligned}$$

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\end{aligned}$$

$$s = 0:$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$

$$\frac{1}{(s^2+1)(3s+2)} = \frac{As+B}{s^2+1} + \frac{C}{3s+2}$$

$$1 = (As+B)(3s+2) + C\left(s^2+1\right)$$

$$s = -\frac{2}{3}: \quad 1 = C\left(\frac{4}{9}+1\right) = \frac{13}{9}C, \quad C = \frac{9}{13}$$

$$s = 0: \quad 1 = B \cdot 2 + C \cdot 1$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$

$$\begin{aligned}
\frac{1}{\left(s^2+1\right)(3s+2)} &= \frac{As+B}{s^2+1} + \frac{C}{3s+2} \\
1 &= (As+B)(3s+2) + C\left(s^2+1\right) \\
s = -\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right) = \frac{13}{9}C, \quad C = \frac{9}{13} \\
s = 0: \quad 1 &= B\cdot 2 + C\cdot 1 = 2B + \frac{9}{13}
\end{aligned}$$

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s = 0: \quad 1 &= B\cdot 2 + C\cdot 1 = 2B + \frac{9}{13}, \quad B = \frac{2}{13}
\end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

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$$s = 0: \quad 1 = B\cdot 2 + C\cdot 1 = 2B + \frac{9}{13}, \quad B = \frac{2}{13}$$

$$s = 1:$$

Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|$, $y(0)=0$

$$\begin{aligned}
\frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1}+\frac{C}{3s+2} \\
1 &= (As+B)(3s+2)+C\left(s^2+1\right) \\
s=-\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right)=\frac{13}{9}C, \quad C=\frac{9}{13} \\
s=0: \quad 1 &= B\cdot 2+C\cdot 1=2B+\frac{9}{13}, \quad B=\frac{2}{13} \\
s=1: \quad 1 &= (A+B)\cdot 5+C\cdot 2
\end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, \; y(0) = 0$

$$\begin{aligned}
\frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1} + \frac{C}{3s+2} \\
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s = -\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right) = \frac{13}{9}C, \quad C = \frac{9}{13} \\
s = 0: \quad 1 &= B\cdot 2 + C\cdot 1 = 2B + \frac{9}{13}, \quad B = \frac{2}{13} \\
s = 1: \quad 1 &= (A+B)\cdot 5 + C\cdot 2 = 5A + \frac{10}{13} + \frac{18}{13}
\end{aligned}$$

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\begin{aligned}
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\end{aligned}
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\frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1}+\frac{C}{3s+2}\\
1 &= (As+B)(3s+2)+C\left(s^2+1\right)\\
s=-\frac{2}{3}:\quad 1 &= C\left(\frac{4}{9}+1\right)=\frac{13}{9}C,\quad C=\frac{9}{13}\\
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s=1:\quad 1 &= (A+B)\cdot 5+C\cdot 2=5A+\frac{10}{13}+\frac{18}{13},\quad A=-\frac{3}{13}
\end{aligned}$$

$$\frac{1}{(s^2+1)(3s+2)}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$\begin{aligned}
\frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1} + \frac{C}{3s+2} \\
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s = 1: \quad 1 &= (A+B)\cdot 5 + C\cdot 2 = 5A + \frac{10}{13} + \frac{18}{13}, \quad A = -\frac{3}{13} \\
\frac{1}{(s^2+1)(3s+2)} &= \frac{1}{13}
\end{aligned}$$

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\frac{1}{\left(s^2+1\right)(3s+2)} &= \frac{As+B}{s^2+1}+\frac{C}{3s+2}\\
1 &= (As+B)(3s+2)+C\left(s^2+1\right)\\
s=-\frac{2}{3}:\quad 1 &= C\left(\frac{4}{9}+1\right)=\frac{13}{9}C,\quad C=\frac{9}{13}\\
s=0:\quad 1 &= B\cdot 2+C\cdot 1=2B+\frac{9}{13},\quad B=\frac{2}{13}\\
s=1:\quad 1 &= (A+B)\cdot 5+C\cdot 2=5A+\frac{10}{13}+\frac{18}{13},\quad A=-\frac{3}{13}\\
\frac{1}{\left(s^2+1\right)(3s+2)} &= \frac{1}{13}\left(\frac{-3s+2}{s^2+1}\right.
\end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|, \ y(0) = 0$

$$\begin{aligned}
\frac{1}{(s^2+1)(3s+2)} &= \frac{As+B}{s^2+1} + \frac{C}{3s+2} \\
1 &= (As+B)(3s+2) + C\left(s^2+1\right) \\
s = -\frac{2}{3}: \quad 1 &= C\left(\frac{4}{9}+1\right) = \frac{13}{9}C, \quad C = \frac{9}{13} \\
s = 0: \quad 1 &= B\cdot 2 + C\cdot 1 = 2B + \frac{9}{13}, \quad B = \frac{2}{13} \\
s = 1: \quad 1 &= (A+B)\cdot 5 + C\cdot 2 = 5A + \frac{10}{13} + \frac{18}{13}, \quad A = -\frac{3}{13} \\
\frac{1}{(s^2+1)(3s+2)} &= \frac{1}{13}\left(\frac{-3s+2}{s^2+1} + \frac{9}{3s+2}\right)
\end{aligned}$$

Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|,\ y(0)=0$

$$\frac{1}{\left(s^2+1\right)\left(3s+2\right)} = \frac{As+B}{s^2+1}+\frac{C}{3s+2}$$

$$1 = (As+B)(3s+2)+C\left(s^2+1\right)$$

$$s=-\frac{2}{3}: \quad 1 = C\left(\frac{4}{9}+1\right)=\frac{13}{9}C, \quad C=\frac{9}{13}$$

$$s=0: \quad 1 = B\cdot 2+C\cdot 1=2B+\frac{9}{13}, \quad B=\frac{2}{13}$$

$$s=1: \quad 1 = (A+B)\cdot 5+C\cdot 2=5A+\frac{10}{13}+\frac{18}{13}, \quad A=-\frac{3}{13}$$

$$\frac{1}{\left(s^2+1\right)\left(3s+2\right)} = \frac{1}{13}\left(\frac{-3s+2}{s^2+1}+\frac{9}{3s+2}\right)$$

$$= \frac{1}{13}\left(-3\frac{s}{s^2+1}+2\frac{1}{s^2+1}+3\frac{1}{s+\frac{2}{3}}\right)$$

Solve the Initial Value Problem

$3y' + 2y = \left|\sin(t)\right|, \ y(0) = 0$

Solve the Initial Value Problem
$3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$Y = \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{(s^2+1)(3s+2)}$$

Solve the Initial Value Problem

$3y' + 2y = \left|\sin(t)\right|, y(0) = 0$

$$
\begin{aligned}
Y &= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{\left(s^2+1\right)(3s+2)} \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right)
\end{aligned}
$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|,\ y(0) = 0$

$$
\begin{aligned}
Y &= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{(s^2+1)(3s+2)} \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right) \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=1}^{\infty} e^{-n\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right)
\end{aligned}
$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|,\ y(0) = 0$

$$
\begin{aligned}
Y &= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{(s^2+1)(3s+2)} \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=0}^{\infty} e^{-(n+1)\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right) \\
&= \left[\sum_{n=0}^{\infty} e^{-n\pi s} + \sum_{n=1}^{\infty} e^{-n\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right) \\
&= \left[1 + 2\sum_{n=1}^{\infty} e^{-n\pi s}\right] \frac{1}{13}\left(-3\frac{s}{s^2+1} + 2\frac{1}{s^2+1} + 3\frac{1}{s+\frac{2}{3}}\right)
\end{aligned}
$$

Solve the Initial Value Problem $3y' + 2y = \big|\sin(t)\big|,\ y(0) = 0$

Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|, y(0)=0$

$$\begin{aligned} y &= \frac{1}{13}\left[-3\cos(t)+2\sin(t)+3e^{-\frac{2}{3}t}\right] \\ &\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}\left(t-n\pi\right)\left[-3\cos(t)+2\sin(t)+3e^{-\frac{2}{3}t}\right]_{t\to t-n\pi} \end{aligned}$$

Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$

$$
\begin{aligned}
y &= \frac{1}{13}\left[-3\cos(t) + 2\sin(t) + 3e^{-\frac{2}{3}t}\right] \\
&\quad + \frac{2}{13}\sum_{n=1}^{\infty} \mathscr{U}(t-n\pi)\left[-3\cos(t) + 2\sin(t) + 3e^{-\frac{2}{3}t}\right]_{t\to t-n\pi} \\
&= \frac{1}{13}\left[-3\cos(t) + 2\sin(t) + 3e^{-\frac{2}{3}t}\right] \\
&\quad + \frac{2}{13}\sum_{n=1}^{\infty} \mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi) + 2\sin(t-n\pi) + 3e^{-\frac{2}{3}(t-n\pi)}\right]
\end{aligned}
$$

Does $y = \frac{1}{13}\left[-3\cos(t)+2\sin(t)+3e^{-\frac{2}{3}t}\right]$
$+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi)+2\sin(t-n\pi)+3e^{-\frac{2}{3}(t-n\pi)}\right]$ Really Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|$, $y(0)=0$?

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$+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi)+2\sin(t-n\pi)+3e^{-\frac{2}{3}(t-n\pi)}\right]$ Really Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|$, $y(0)=0$?

$y(0)$

Does $y = \frac{1}{13}\left[-3\cos(t) + 2\sin(t) + 3e^{-\frac{2}{3}t}\right] + \frac{2}{13}\sum_{n=1}^{\infty} \mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi) + 2\sin(t-n\pi) + 3e^{-\frac{2}{3}(t-n\pi)}\right]$ Really Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$?

$$\begin{aligned} y(0) &= \frac{1}{13}\left[-3\cos(0) + 2\sin(0) + 3e^{-\frac{2}{3}0}\right] \\ &\quad + \frac{2}{13}\sum_{n=1}^{\infty} \mathscr{U}(0-n\pi)\left[-3\cos(0-n\pi) + 2\sin(0-n\pi) + 3e^{-\frac{2}{3}(0-n\pi)}\right] \end{aligned}$$

Does $y = \frac{1}{13}\left[-3\cos(t)+2\sin(t)+3e^{-\frac{2}{3}t}\right] + \frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi)+2\sin(t-n\pi)+3e^{-\frac{2}{3}(t-n\pi)}\right]$ Really Solve the Initial Value Problem $3y'+2y=\left|\sin(t)\right|$, $y(0)=0$?

$$\begin{aligned} y(0) &= \frac{1}{13}\left[-3\cos(0)+2\sin(0)+3e^{-\frac{2}{3}0}\right] \\ &\quad + \frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(0-n\pi)\left[-3\cos(0-n\pi)+2\sin(0-n\pi)+3e^{-\frac{2}{3}(0-n\pi)}\right] \\ &= 0 \end{aligned}$$

Does $y = \frac{1}{13}\left[-3\cos(t) + 2\sin(t) + 3e^{-\frac{2}{3}t}\right]$
$+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-3\cos(t-n\pi) + 2\sin(t-n\pi) + 3e^{-\frac{2}{3}(t-n\pi)}\right]$ Really Solve the Initial Value Problem $3y' + 2y = \left|\sin(t)\right|$, $y(0) = 0$?

$$
\begin{aligned}
y(0) &= \frac{1}{13}\left[-3\cos(0) + 2\sin(0) + 3e^{-\frac{2}{3}0}\right] \\
&\quad + \frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(0-n\pi)\left[-3\cos(0-n\pi) + 2\sin(0-n\pi) + 3e^{-\frac{2}{3}(0-n\pi)}\right] \\
&= 0 \qquad \surd
\end{aligned}
$$

$$
\begin{aligned}
y' &= \frac{1}{13}\left[3\sin(t)+2\cos(t)-2e^{-\frac{2}{3}t}\right]\\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[3\sin(t-n\pi)+2\cos(t-n\pi)-2e^{-\frac{2}{3}(t-n\pi)}\right]
\end{aligned}
$$

$$
\begin{aligned}
y' &= \frac{1}{13}\left[3\sin(t)+2\cos(t)-2e^{-\frac{2}{3}t}\right] \\
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3y'+2y &= \frac{1}{13}\left[9\sin(t)+6\cos(t)-6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+6\cos(t-n\pi)-6e^{-\frac{2}{3}(t-n\pi)}\right]
\end{aligned}
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&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+6\cos(t-n\pi)-6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&\quad +\frac{1}{13}\left[-6\cos(t)+4\sin(t)+6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-6\cos(t-n\pi)+4sin(t-n\pi)+6e^{-\frac{2}{3}(t-n\pi)}\right]
\end{aligned}$$

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\begin{aligned}
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&\quad +\frac{1}{13}\left[-6\cos(t)+4\sin(t)+6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-6\cos(t-n\pi)+4sin(t-n\pi)+6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&= \frac{1}{13}\left[9\sin(t)+4\sin(t)\right]+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+4\sin(t-n\pi)\right]
\end{aligned}
$$

$$
\begin{aligned}
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3y'+2y &= \frac{1}{13}\left[9\sin(t)+6\cos(t)-6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+6\cos(t-n\pi)-6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&\quad +\frac{1}{13}\left[-6\cos(t)+4\sin(t)+6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-6\cos(t-n\pi)+4sin(t-n\pi)+6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&= \frac{1}{13}\left[9\sin(t)+4\sin(t)\right]+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+4\sin(t-n\pi)\right] \\
&= \sin(t)+2\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\sin(t-n\pi)
\end{aligned}
$$

$$
\begin{aligned}
y' &= \frac{1}{13}\left[3\sin(t)+2\cos(t)-2e^{-\frac{2}{3}t}\right] \\
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&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+6\cos(t-n\pi)-6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&\quad +\frac{1}{13}\left[-6\cos(t)+4\sin(t)+6e^{-\frac{2}{3}t}\right] \\
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-6\cos(t-n\pi)+4sin(t-n\pi)+6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&= \frac{1}{13}\left[9\sin(t)+4\sin(t)\right]+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+4\sin(t-n\pi)\right] \\
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$$
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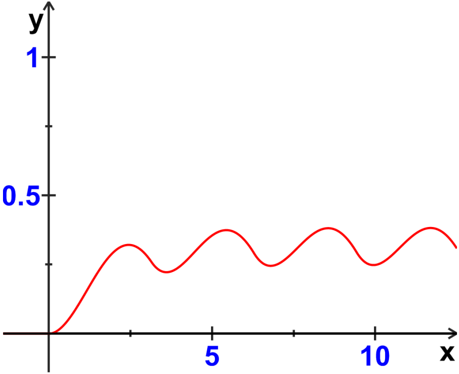
&\quad +\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[-6\cos(t-n\pi)+4sin(t-n\pi)+6e^{-\frac{2}{3}(t-n\pi)}\right] \\
&= \frac{1}{13}\left[9\sin(t)+4\sin(t)\right]+\frac{2}{13}\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\left[9\sin(t-n\pi)+4\sin(t-n\pi)\right] \\
&= \sin(t)+2\sum_{n=1}^{\infty}\mathscr{U}(t-n\pi)\sin(t-n\pi)=\left|\sin(t)\right| \qquad \surd
\end{aligned}
$$

Comparing Output to Input

Comparing Output to Input



Comparing Output to Input