Laplace Transforms for Systems of Differential Equations

Bernd Schröder

The Laplace Transform of a System

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The Laplace Transform of a System

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4. Transform back.
5. The example will be first order, but the idea works for any order.

So Everything Remains As It Was

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Time Domain (t)

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Original
DE & IVP

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Time Domain (t)

$$\boxed{\begin{array}{c}\text{Original}\\ \text{DE \& IVP}\end{array}} \xrightarrow{\mathscr{L}}$$

So Everything Remains As It Was

Time Domain (t)

Original DE & IVP	$\xrightarrow{\mathscr{L}}$	Algebraic equation for the Laplace transform

So Everything Remains As It Was

$$\begin{array}{ccc} \text{Time Domain } (t) & & \text{Transform domain } (s) \\ \boxed{\begin{array}{c}\text{Original}\\ \text{DE \& IVP}\end{array}} & \xrightarrow{\mathscr{L}} & \boxed{\begin{array}{c}\text{Algebraic equation for}\\ \text{the Laplace transform}\end{array}} \end{array}$$

So Everything Remains As It Was

Time Domain (t)

Transform domain (s)

Original DE & IVP $\xrightarrow{\mathscr{L}}$ Algebraic equation for the Laplace transform

Algebraic solution, partial fractions

So Everything Remains As It Was

Time Domain (t)

Transform domain (s)

Original
DE & IVP

$\xrightarrow{\mathscr{L}}$

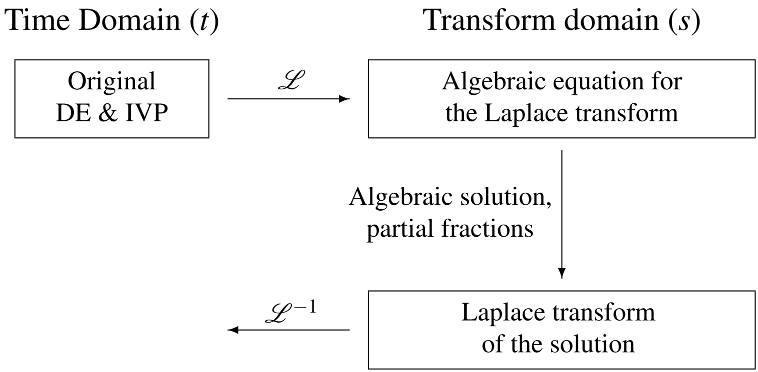
Algebraic equation for
the Laplace transform

Algebraic solution,
partial fractions

$\downarrow$

Laplace transform
of the solution

So Everything Remains As It Was



So Everything Remains As It Was

$$\begin{tikzcd} \text{Time Domain } (t) & \text{Transform domain } (s) \\ \boxed{\text{Original DE \& IVP}} \arrow[r, "\mathscr{L}"] & \boxed{\text{Algebraic equation for the Laplace transform}} \arrow[d, "\text{Algebraic solution, partial fractions}"'] \\ \boxed{\text{Solution}} & \boxed{\text{Laplace transform of the solution}} \arrow[l, "\mathscr{L}^{-1}"'] \end{tikzcd}$$

Solve the Initial Value Problem

$6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

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3. ... but the second equation also relates the initial values to each other. So certain combinations of initial values will not be possible.

Solve the Initial Value Problem

$$6x + 6y' + y = 2e^{-t},\ 2x - y = 0,\ x(0) = 1,\ y(0) = 2$$

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4. The initial values in this example are allowed.

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2. This is not a problem. The transforms will work the same way, ...
3. ... but the second equation also relates the initial values to each other. So certain combinations of initial values will not be possible.
4. The initial values in this example are allowed.
5. The example itself is related to equations that come from the analysis of two loop circuits. So systems such as this one certainly arise in applications.

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$$\begin{aligned} 6x+6y'+y &= 2e^{-t}, \qquad x(0)=1 \\ 2x-y &= 0, \qquad y(0)=2 \\ 6X+6sY-12+Y &= \frac{2}{s+1} \end{aligned}$$

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2X-Y &= 0 \qquad\qquad (\times -3 \text{ and add})
\end{aligned}
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$$Y$$

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$$Y(6s$$

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$$Y(6s+4)$$

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6X+6sY-12+Y &= \frac{2}{s+1} \\
2X-Y &= 0 \qquad\qquad (\times -3 \text{ and add}) \\
Y(6s+4) &= \frac{2}{s+1}+12
\end{aligned}$$

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6X+6sY-12+Y &= \frac{2}{s+1} \\
2X-Y &= 0 \qquad\qquad (\times -3 \text{ and add}) \\
Y(6s+4) &= \frac{2}{s+1}+12=\frac{12s+14}{s+1}
\end{aligned}$$

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2x-y &= 0, \qquad y(0)=2\\
6X+6sY-12+Y &= \frac{2}{s+1}\\
2X-Y &= 0 \qquad\qquad (\times -3 \text{ and add})\\
Y(6s+4) &= \frac{2}{s+1}+12=\frac{12s+14}{s+1}\\
Y &= \frac{12s+14}{(s+1)(6s+4)}
\end{aligned}$$

Solve the Initial Value Problem

$6x + 6y' + y = 2e^{-t}$, $2x - y = 0$, $x(0) = 1$, $y(0) = 2$

Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

$$\frac{12s+14}{(s+1)(6s+4)} = \frac{A}{s+1}+\frac{B}{6s+4}$$

Solve the Initial Value Problem
$6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

$$\begin{aligned}\frac{12s+14}{(s+1)(6s+4)} &= \frac{A}{s+1}+\frac{B}{6s+4}\\ 12s+14 &= A(6s+4)+B(s+1)\end{aligned}$$

Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

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$$s=-1:$$

Solve the Initial Value Problem

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\frac{12s+14}{(s+1)(6s+4)} &= \frac{A}{s+1}+\frac{B}{6s+4} \\
12s+14 &= A(6s+4)+B(s+1) \\
s=-1: \quad 2
\end{aligned}$$

Solve the Initial Value Problem

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\frac{12s+14}{(s+1)(6s+4)} &= \frac{A}{s+1}+\frac{B}{6s+4} \\
12s+14 &= A(6s+4)+B(s+1) \\
s=-1: \quad 2 &= A(-2)
\end{aligned}$$

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\frac{12s+14}{(s+1)(6s+4)} &= \frac{A}{s+1}+\frac{B}{6s+4} \\
12s+14 &= A(6s+4)+B(s+1) \\
s=-1: \quad 2 &= A(-2), \qquad A=-1
\end{aligned}$$

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12s+14 &= A(6s+4)+B(s+1) \\
s=-1: \quad 2 &= A(-2), \qquad A=-1 \\
s=-\frac{2}{3}:
\end{aligned}
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\end{aligned}
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12s+14 &= A(6s+4)+B(s+1) \\
s=-1: \quad 2 &= A(-2), \qquad A=-1 \\
s=-\frac{2}{3}: \quad 6 &= B\left(\frac{1}{3}\right), \qquad B=18
\end{aligned}
$$

Solve the Initial Value Problem

$6x + 6y' + y = 2e^{-t}$, $2x - y = 0$, $x(0) = 1$, $y(0) = 2$

$$
\begin{aligned}
\frac{12s+14}{(s+1)(6s+4)} &= \frac{A}{s+1} + \frac{B}{6s+4} \\
12s+14 &= A(6s+4) + B(s+1) \\
s=-1: \quad 2 &= A(-2), \qquad A = -1 \\
s=-\frac{2}{3}: \quad 6 &= B\left(\frac{1}{3}\right), \qquad B = 18 \\
Y(s) &= -\frac{1}{s+1} + \frac{18}{6s+4}
\end{aligned}
$$

Solve the Initial Value Problem

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Y(s) &= -\frac{1}{s+1}+\frac{18}{6s+4}=-\frac{1}{s+1}+3\frac{1}{s+\frac{2}{3}}
\end{aligned}
$$

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s=-1:\quad 2 &= A(-2), \qquad A=-1\\
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Y(s) &= -\frac{1}{s+1}+\frac{18}{6s+4}=-\frac{1}{s+1}+3\frac{1}{s+\frac{2}{3}}\\
y(t) &= -e^{-t}+3e^{-\frac{2}{3}t}
\end{aligned}
$$

Solve the Initial Value Problem

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$$2X-Y = 0$$

Solve the Initial Value Problem

$6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

$$\begin{aligned} 2X-Y &= 0 \\ X &= \frac{1}{2}Y \end{aligned}$$

Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

$$
\begin{aligned}
2X-Y &= 0\\
X &= \frac{1}{2}Y\\
&= \frac{1}{2}\left[-\frac{1}{s+1}+3\frac{1}{s+\frac{2}{3}}\right]
\end{aligned}
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\begin{aligned}
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Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

$$\begin{aligned}
2X-Y &= 0\\
X &= \frac{1}{2}Y\\
&= \frac{1}{2}\left[-\frac{1}{s+1}+3\frac{1}{s+\frac{2}{3}}\right]\\
&= -\frac{1}{2}\frac{1}{s+1}+\frac{3}{2}\frac{1}{s+\frac{2}{3}}\\
x(t) &= -\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}
\end{aligned}$$

Does $x(t) = -\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}$, $y(t) = -e^{-t} + 3e^{-\frac{2}{3}t}$
Really Solve the Initial Value Problem
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Initial values: Look at x and y!

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$2x-y=0$: Same as $x=\frac{y}{2}$.

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Initial values: Look at x and y!

$2x - y = 0$: Same as $x = \frac{y}{2}$. Look at x and y!

$$6x + 6y' + y = 6\left[-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}\right]$$

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Initial values: Look at x and y!

$2x - y = 0$: Same as $x = \frac{y}{2}$. Look at x and y!

$$6x + 6y' + y = 6\left[-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}\right] + 6\left[e^{-t} - 2e^{-\frac{2}{3}t}\right]$$

Does $x(t) = -\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}$, $y(t) = -e^{-t} + 3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x + 6y' + y = 2e^{-t}$, $2x - y = 0$, $x(0) = 1$, $y(0) = 2$

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$$\begin{aligned} 6x + 6y' + y &= 6\left[-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}\right] + 6\left[e^{-t} - 2e^{-\frac{2}{3}t}\right] \\ &\quad + \left[-e^{-t} + 3e^{-\frac{2}{3}t}\right] \end{aligned}$$

Does $x(t)=-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}$, $y(t)=-e^{-t}+3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

Initial values: Look at x and y!

$2x-y=0$: Same as $x=\frac{y}{2}$. Look at x and y!

$$\begin{aligned} 6x+6y'+y &= 6\left[-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}\right]+6\left[e^{-t}-2e^{-\frac{2}{3}t}\right] \\ &\quad +\left[-e^{-t}+3e^{-\frac{2}{3}t}\right] \\ &= e^{-t} \end{aligned}$$

Does $x(t) = -\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}$, $y(t) = -e^{-t} + 3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x + 6y' + y = 2e^{-t}$, $2x - y = 0$, $x(0) = 1$, $y(0) = 2$

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$$
\begin{aligned}
6x + 6y' + y &= 6\left[-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-\frac{2}{3}t}\right] + 6\left[e^{-t} - 2e^{-\frac{2}{3}t}\right] \\
&\quad + \left[-e^{-t} + 3e^{-\frac{2}{3}t}\right] \\
&= e^{-t}\big(-3
\end{aligned}
$$

Does $x(t)=-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}$, $y(t)=-e^{-t}+3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

Initial values: Look at x and y!

$2x-y=0$: Same as $x=\frac{y}{2}$. Look at x and y!

$$\begin{aligned} 6x+6y'+y &= 6\left[-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}\right]+6\left[e^{-t}-2e^{-\frac{2}{3}t}\right] \\ &\quad +\left[-e^{-t}+3e^{-\frac{2}{3}t}\right] \\ &= e^{-t}\big(-3+6 \end{aligned}$$

Does $x(t)=-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}$, $y(t)=-e^{-t}+3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

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$$\begin{aligned} 6x+6y'+y &= 6\left[-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}\right]+6\left[e^{-t}-2e^{-\frac{2}{3}t}\right] \\ &\quad +\left[-e^{-t}+3e^{-\frac{2}{3}t}\right] \\ &= e^{-t}\left(-3+6-1\right) \end{aligned}$$

Does $x(t)=-\frac{1}{2}e^{-t}+\frac{3}{2}e^{-\frac{2}{3}t}$, $y(t)=-e^{-t}+3e^{-\frac{2}{3}t}$ Really Solve the Initial Value Problem $6x+6y'+y=2e^{-t}$, $2x-y=0$, $x(0)=1$, $y(0)=2$

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\end{aligned}$$