Using Laplace Transforms to Solve Initial Value Problems

Bernd Schröder

How Laplace Transforms Turn Initial Value Problems Into Algebraic Equations

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Time Domain (t)

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Original DE & IVP

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Original DE & IVP	$\xrightarrow{\mathscr{L}}$	Algebraic equation for the Laplace transform

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Time Domain (t) | Transform domain (s)

Original DE & IVP $\xrightarrow{\mathscr{L}}$ Algebraic equation for the Laplace transform

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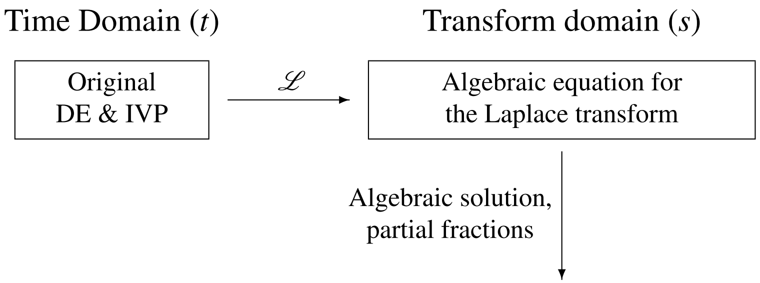

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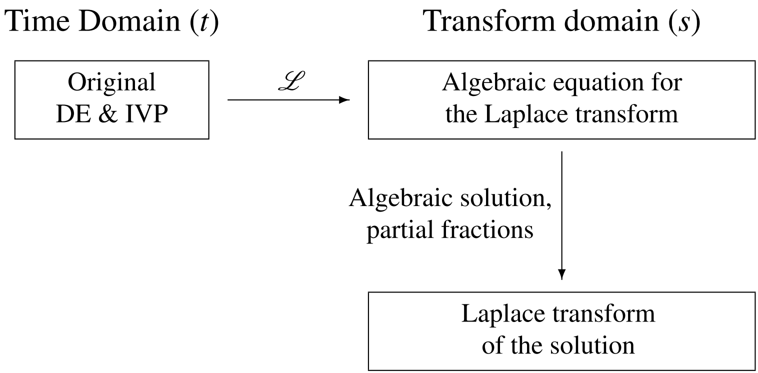

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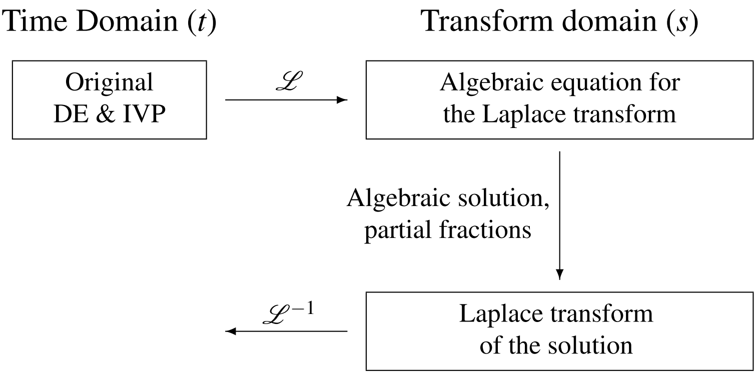

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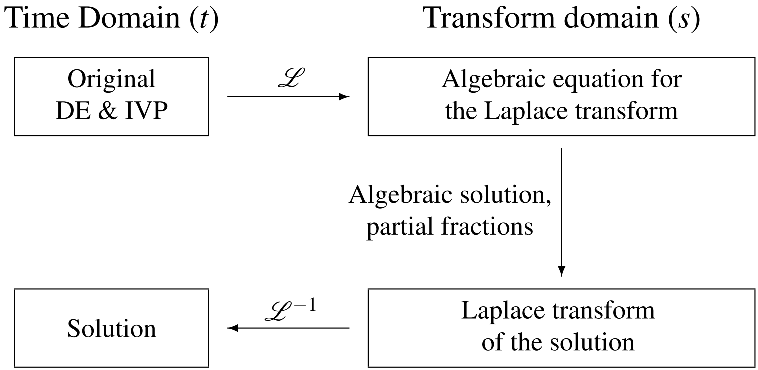

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2. The right sides above do not involve derivatives of whatever Y is.
3. The other key property is that constants and sums "factor through" the Laplace transform:
   $\mathscr{L}\{f+g\} = \mathscr{L}\{f\} + \mathscr{L}\{g\}$ and $\mathscr{L}\{af\} = a\mathscr{L}\{f\}$.
   (That is, the Laplace transform is linear.)

Solve the Initial Value Problem

$y'' + 7y' + 12y = 0,\ y(0) = 1,\ y'(0) = 2$

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$$s^2Y-s-2+7sY-7$$

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Finding the Laplace transform of the solution.

$$\begin{aligned} y''+7y'+12y &= 0,\ y(0)=1,\ y'(0)=2 \\ s^2Y-s-2+7sY-7+12Y &= 0 \end{aligned}$$

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\begin{aligned}
y'' + 7y' + 12y &= 0,\ y(0) = 1,\ y'(0) = 2 \\
s^2Y - s - 2 + 7sY - 7 + 12Y &= 0 \\
\left(s^2 + 7s + 12\right)Y &
\end{aligned}
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\left(s^2 + 7s + 12\right)Y &= s + 9 \\
Y &= \frac{s+9}{s^2 + 7s + 12}
\end{aligned}$$

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Y &= \frac{s+9}{s^2+7s+12}\\
&= \frac{s+9}{(s+3)(s+4)}
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Partial fraction decomposition.

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$$Y = \frac{s+9}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4}$$

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s=-3 & \quad A=6
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&\quad \text{Heaviside's Method :} \\
s = -3 &\quad A = 6 \\
s = -4 &\quad B = -5 \\
Y &= \frac{6}{s+3} - \frac{5}{s+4}
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Use the transform table.

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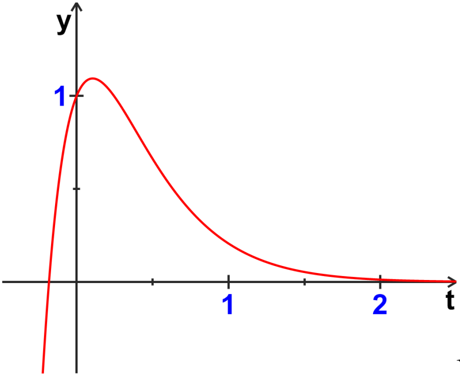

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\begin{aligned}
y'' + 7y' + 12y &\stackrel{?}{=} 0 \\
\left(54e^{-3t} - 80e^{-4t}\right) + 7\left(-18e^{-3t} + 20e^{-4t}\right) + 12\left(6e^{-3t} - 5e^{-4t}\right) &\stackrel{?}{=} 0
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