Variation of Parameters

Bernd Schröder

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2. Variation of Parameters is a way to obtain a particular solution of the inhomogeneous equation.
3. The particular solution can be obtained as follows.
   3.1 Assume that the parameters in the solution of the homogeneous equation are functions. (Hence the name.)
   3.2 Substitute the expression into the inhomogeneous equation and solve for the parameters.

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$$y_p(x) = -y_1(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt + y_2(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt,$$

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where y_1, y_2 are linearly independent solutions of the homogeneous equation and $W(y_1,y_2) := y_1y_2' - y_2y_1'$.

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$W(y_1,y_2) = \det\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$.

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where y_1, y_2 are linearly independent solutions of the homogeneous equation and $W(y_1,y_2) := y_1y_2' - y_2y_1'$.
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The Wronskian can also be represented as
$W(y_1,y_2) = \det\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$.
That's it.

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

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$$y''+4y'+4y = 0$$

Solve the Initial Value Problem $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Solution of the homogeneous equation.

$$\begin{aligned} y'' + 4y' + 4y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \end{aligned}$$

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Solution of the homogeneous equation.

$$\begin{aligned} y'' + 4y' + 4y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda + 4 &= 0 \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Solution of the homogeneous equation.

$$\begin{aligned} y''+4y'+4y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0 \\ \lambda^2+4\lambda+4 &= 0 \\ \lambda_{1,2} &= \frac{-4\pm\sqrt{16-16}}{2} \end{aligned}$$

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Solution of the homogeneous equation.

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Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Solution of the homogeneous equation.

$$\begin{aligned}
y''+4y'+4y &= 0\\
\lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0\\
\lambda^2+4\lambda+4 &= 0\\
\lambda_{1,2} &= \frac{-4\pm\sqrt{16-16}}{2}=-2\\
y_h &= c_1e^{-2x}+c_2xe^{-2x}
\end{aligned}$$

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Computing the Wronskian.

$$W(y_1, y_2)(t) = e^{-2t}\left(e^{-2t} - 2te^{-2t}\right) - te^{-2t}\left(-2e^{-2t}\right)$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Computing the Wronskian.

$$\begin{aligned} W(y_1,y_2)(t) &= e^{-2t}\left(e^{-2t}-2te^{-2t}\right)-te^{-2t}\left(-2e^{-2t}\right) \\ &= e^{-4t}-2te^{-4t}+2te^{-4t} \end{aligned}$$

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Computing the integrals.

$$\int \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_1(t)\, dt$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Computing the integrals.

$$\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt \quad = \quad \int \frac{1}{e^{-4t}}\frac{\sin(t)}{1}e^{-2t}\,dt$$

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Computing the integrals.

$$\begin{aligned}\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt &= \int \frac{1}{e^{-4t}}\frac{\sin(t)}{1}e^{-2t}\,dt\\ &= \int e^{2t}\sin(t)\,dt\end{aligned}$$

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Computing the integrals.

$$\int e^{2t} \sin(t)\, dt$$

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$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Computing the integrals.

$$\int e^{2t}\sin(t)\,dt = \frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt$$

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Computing the integrals.

$$\begin{aligned}\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt\\ &= \frac{1}{2}e^{2t}\sin(t)-\frac{1}{2}\left[\frac{1}{2}e^{2t}\cos(t)-\int \frac{1}{2}e^{2t}\big(-\sin(t)\big)\,dt\right]\end{aligned}$$

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Computing the integrals.

$$\begin{aligned}
\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t) - \int \frac{1}{2}e^{2t}\cos(t)\,dt \\
&= \frac{1}{2}e^{2t}\sin(t) - \frac{1}{2}\left[\frac{1}{2}e^{2t}\cos(t) - \int \frac{1}{2}e^{2t}\big(-\sin(t)\big)\,dt\right] \\
&= \frac{1}{2}e^{2t}\sin(t) - \frac{1}{4}e^{2t}\cos(t) - \frac{1}{4}\int e^{2t}\sin(t)\,dt
\end{aligned}$$

Solve the Initial Value Problem

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Computing the integrals.

$$\begin{aligned}
\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt\\
&= \frac{1}{2}e^{2t}\sin(t)-\frac{1}{2}\left[\frac{1}{2}e^{2t}\cos(t)-\int \frac{1}{2}e^{2t}\big(-\sin(t)\big)\,dt\right]\\
&= \frac{1}{2}e^{2t}\sin(t)-\frac{1}{4}e^{2t}\cos(t)-\frac{1}{4}\int e^{2t}\sin(t)\,dt\\
\frac{5}{4}\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t)-\frac{1}{4}e^{2t}\cos(t)
\end{aligned}$$

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Computing the integrals.

$$\begin{aligned}
\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t) - \int \frac{1}{2}e^{2t}\cos(t)\,dt \\
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\frac{5}{4}\int e^{2t}\sin(t)\,dt &= \frac{1}{2}e^{2t}\sin(t) - \frac{1}{4}e^{2t}\cos(t) \\
\int e^{2t}\sin(t)\,dt &= \frac{2}{5}e^{2t}\sin(t) - \frac{1}{5}e^{2t}\cos(t)
\end{aligned}$$

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Computing the integrals.

$$\int \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_2(t)\, dt$$

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Computing the integrals.

$$\int \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_2(t)\, dt$$

$$= \int \frac{1}{e^{-4t}} \frac{\sin(t)}{1} te^{-2t}\, dt$$

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Computing the integrals.

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$$= \int \frac{1}{e^{-4t}}\frac{\sin(t)}{1}te^{-2t}\,dt$$

$$= \int te^{2t}\sin(t)\,dt$$

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Computing the integrals.

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$$= \int te^{2t}\sin(t)\,dt$$

$$= t\left(\frac{2}{5}e^{2t}\sin(t)-\frac{1}{5}e^{2t}\cos(t)\right)-\int \frac{2}{5}e^{2t}\sin(t)-\frac{1}{5}e^{2t}\cos(t)\,dt$$

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$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\int e^{2t}\sin(t)\,dt+\frac{1}{5}\int e^{2t}\cos(t)\,dt$$

Solve the Initial Value Problem

$$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Computing the integrals.

$$\int e^{2t}\cos(t)\,dt$$

Solve the Initial Value Problem
$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Computing the integrals.

$$\int e^{2t}\cos(t)\,dt = \frac{1}{2}e^{2t}\cos(t) + \int \frac{1}{2}e^{2t}\sin(t)\,dt$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Computing the integrals.

$$\begin{aligned}
\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t)+\int \frac{1}{2}e^{2t}\sin(t)\,dt \\
&= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{2}\left[\frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt\right]
\end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Computing the integrals.

$$\begin{aligned}
\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t)+\int \frac{1}{2}e^{2t}\sin(t)\,dt \\
&= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{2}\left[\frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt\right] \\
&= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{4}e^{2t}\sin(t)-\frac{1}{4}\int e^{2t}\cos(t)\,dt
\end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Computing the integrals.

$$\begin{aligned}
\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t)+\int \frac{1}{2}e^{2t}\sin(t)\,dt \\
&= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{2}\left[\frac{1}{2}e^{2t}\sin(t)-\int \frac{1}{2}e^{2t}\cos(t)\,dt\right] \\
&= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{4}e^{2t}\sin(t)-\frac{1}{4}\int e^{2t}\cos(t)\,dt \\
\frac{5}{4}\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t)+\frac{1}{4}e^{2t}\sin(t)
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Computing the integrals.

$$\begin{aligned}
\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t) + \int \frac{1}{2}e^{2t}\sin(t)\,dt \\
&= \frac{1}{2}e^{2t}\cos(t) + \frac{1}{2}\left[\frac{1}{2}e^{2t}\sin(t) - \int \frac{1}{2}e^{2t}\cos(t)\,dt\right] \\
&= \frac{1}{2}e^{2t}\cos(t) + \frac{1}{4}e^{2t}\sin(t) - \frac{1}{4}\int e^{2t}\cos(t)\,dt \\
\frac{5}{4}\int e^{2t}\cos(t)\,dt &= \frac{1}{2}e^{2t}\cos(t) + \frac{1}{4}e^{2t}\sin(t) \\
\int e^{2t}\cos(t)\,dt &= \frac{2}{5}e^{2t}\cos(t) + \frac{1}{5}e^{2t}\sin(t)
\end{aligned}$$

Solve the Initial Value Problem

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Computing the integrals.

$$\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt$$

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Computing the integrals.

$$\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt$$

$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\int e^{2t}\sin(t)\,dt+\frac{1}{5}\int e^{2t}\cos(t)\,dt$$

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Computing the integrals.

$$\begin{aligned}
&\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt\\
&= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\int e^{2t}\sin(t)\,dt+\frac{1}{5}\int e^{2t}\cos(t)\,dt\\
&= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\left[\frac{2}{5}e^{2t}\sin(t)-\frac{1}{5}e^{2t}\cos(t)\right]
\end{aligned}$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Computing the integrals.

$$\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt$$

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$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\left[\frac{2}{5}e^{2t}\sin(t)-\frac{1}{5}e^{2t}\cos(t)\right]$$

$$+\frac{1}{5}\left[\frac{2}{5}e^{2t}\cos(t)+\frac{1}{5}e^{2t}\sin(t)\right]$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Computing the integrals.

$$\int \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt$$

$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\int e^{2t}\sin(t)\,dt+\frac{1}{5}\int e^{2t}\cos(t)\,dt$$

$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{2}{5}\left[\frac{2}{5}e^{2t}\sin(t)-\frac{1}{5}e^{2t}\cos(t)\right]$$

$$+\frac{1}{5}\left[\frac{2}{5}e^{2t}\cos(t)+\frac{1}{5}e^{2t}\sin(t)\right]$$

$$= \frac{2}{5}te^{2t}\sin(t)-\frac{1}{5}te^{2t}\cos(t)-\frac{3}{25}e^{2t}\sin(t)+\frac{4}{25}e^{2t}\cos(t)$$

Solve the Initial Value Problem

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Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Stating the general solution.

Solve the Initial Value Problem $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Stating the general solution.

$$y_p = -y_1(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt + y_2(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Stating the general solution.

$$\begin{aligned} y_p &= -y_1(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt+y_2(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt \\ &= -e^{-2x}\left[\frac{2}{5}xe^{2x}\sin(x)-\frac{1}{5}xe^{2x}\cos(x)-\frac{3}{25}e^{2x}\sin(x)+\frac{4}{25}e^{2x}\cos(x)\right] \end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Stating the general solution.

$$
\begin{aligned}
y_p &= -y_1(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)} y_2(t)\,dt + y_2(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)} y_1(t)\,dt \\
&= -e^{-2x}\left[\frac{2}{5}xe^{2x}\sin(x) - \frac{1}{5}xe^{2x}\cos(x) - \frac{3}{25}e^{2x}\sin(x) + \frac{4}{25}e^{2x}\cos(x)\right] \\
&\quad + xe^{-2x}\left[\frac{2}{5}e^{2x}\sin(x) - \frac{1}{5}e^{2x}\cos(x)\right]
\end{aligned}
$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Stating the general solution.

$$\begin{aligned} y_p &= -y_1(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt + y_2(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt \\ &= -e^{-2x}\left[\frac{2}{5}xe^{2x}\sin(x)-\frac{1}{5}xe^{2x}\cos(x)-\frac{3}{25}e^{2x}\sin(x)+\frac{4}{25}e^{2x}\cos(x)\right] \\ &\quad +xe^{-2x}\left[\frac{2}{5}e^{2x}\sin(x)-\frac{1}{5}e^{2x}\cos(x)\right] \\ &= \frac{3}{25}\sin(x)-\frac{4}{25}\cos(x) \end{aligned}$$

Solve the Initial Value Problem
$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Stating the general solution.

$$\begin{aligned}
y_p &= -y_1(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt+y_2(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt\\
&= -e^{-2x}\left[\frac{2}{5}xe^{2x}\sin(x)-\frac{1}{5}xe^{2x}\cos(x)-\frac{3}{25}e^{2x}\sin(x)+\frac{4}{25}e^{2x}\cos(x)\right]\\
&\quad +xe^{-2x}\left[\frac{2}{5}e^{2x}\sin(x)-\frac{1}{5}e^{2x}\cos(x)\right]\\
&= \frac{3}{25}\sin(x)-\frac{4}{25}\cos(x)\\
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x}
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$y = -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$
\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\
y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right)
\end{aligned}
$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned} y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\ y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right) \\ 1 &= y(0) \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x}\\
y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right)\\
1 &= y(0)=-\frac{4}{25}+c_1
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned} y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\ y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\ 1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \end{aligned}$$

Solve the Initial Value Problem
$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Finding c_1, c_2.

$$
\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \\
0 &= y'(0)
\end{aligned}
$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x}\\
y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right)\\
1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25}\\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Finding c_1, c_2.

$$
\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \\
0 &= y'(0) = \frac{3}{25} - 2c_1 + c_2, \qquad c_2 = 2c_1 - \frac{3}{25}
\end{aligned}
$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \\
0 &= y'(0) = \frac{3}{25} - 2c_1 + c_2, \qquad c_2 = 2c_1 - \frac{3}{25} = \frac{55}{25}
\end{aligned}$$

Solve the Initial Value Problem
$y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\
y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right) \\
1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25} \\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2, \qquad c_2=2c_1-\frac{3}{25}=\frac{55}{25}=\frac{11}{5}
\end{aligned}$$

Solve the Initial Value Problem

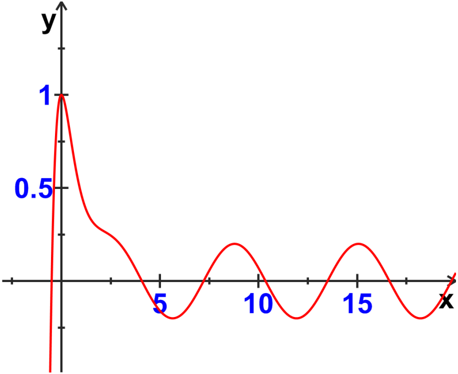
$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \\
0 &= y'(0) = \frac{3}{25} - 2c_1 + c_2, \qquad c_2 = 2c_1 - \frac{3}{25} = \frac{55}{25} = \frac{11}{5} \\
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$



$$y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)$$
$$+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} & 4\left(-\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}\right) \\ & +4\left(\frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{11}{5}\left(e^{-2x} - 2xe^{-2x}\right)\right) \\ & +\frac{4}{25}\cos(x) - \frac{3}{25}\sin(x) + \frac{116}{25}e^{-2x} + \frac{11}{5}\left(-4e^{-2x} + 4xe^{-2x}\right) \quad \stackrel{?}{=} \quad \sin(x) \end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)$$
$$+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)$$
$$+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)$$
$$\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)\\
&+\left(\frac{116}{25}-\frac{232}{25}+\frac{44}{5}+\frac{116}{25}-\frac{44}{5}\right)e^{-2x}
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \quad \stackrel{?}{=} \quad \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)\\
&+\left(\frac{116}{25}-\frac{232}{25}+\frac{44}{5}+\frac{116}{25}-\frac{44}{5}\right)e^{-2x}+\left(\frac{44}{5}-\frac{88}{5}+\frac{44}{5}\right)xe^{-2x}
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)\\
&+\left(\frac{116}{25}-\frac{232}{25}+\frac{44}{5}+\frac{116}{25}-\frac{44}{5}\right)e^{-2x}+\left(\frac{44}{5}-\frac{88}{5}+\frac{44}{5}\right)xe^{-2x} \stackrel{?}{=} \sin(x)
\end{aligned}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
& 4\left(-\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}\right) \\
& +4\left(\frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{11}{5}\left(e^{-2x} - 2xe^{-2x}\right)\right) \\
& +\frac{4}{25}\cos(x) - \frac{3}{25}\sin(x) + \frac{116}{25}e^{-2x} + \frac{11}{5}\left(-4e^{-2x} + 4xe^{-2x}\right) \stackrel{?}{=} \sin(x) \\
& \left(-\frac{16}{25} + \frac{12}{25} + \frac{4}{25}\right)\cos(x) + \left(\frac{12}{25} + \frac{16}{25} - \frac{3}{25}\right)\sin(x) \\
& +\left(\frac{116}{25} - \frac{232}{25} + \frac{44}{5} + \frac{116}{25} - \frac{44}{5}\right)e^{-2x} + \left(\frac{44}{5} - \frac{88}{5} + \frac{44}{5}\right)xe^{-2x} \stackrel{\surd}{=} \sin(x)
\end{aligned}
$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$y(0) = -\frac{4}{25} + \frac{29}{25}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$y(0) = -\frac{4}{25}+\frac{29}{25}=1$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$y(0) = -\frac{4}{25}+\frac{29}{25}=1 \quad \surd$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\ y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \end{aligned}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\ y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\ y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25} \end{aligned}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\
y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\
y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25} = 0
\end{aligned}
$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned}
y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\
y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\
y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25} = 0 \quad \surd
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
y(0) &= -\frac{4}{25}+\frac{29}{25}=1 \quad \surd \\
y'(x) &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{55}{25}\left(e^{-2x}-2xe^{-2x}\right) \\
y'(0) &= \frac{3}{25}-\frac{58}{25}+\frac{55}{25}=0 \quad \surd
\end{aligned}$$

Alternatively, use a computer to double check the result.

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\ y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\ y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25} = 0 \quad \surd \end{aligned}$$

Alternatively, use a computer to double check the result. Beyond a certain level of complexity, that really is the way to go.

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5. The simple solutions of toy problems are best forgotten.
6. We now have a tool that can handle real life situations.