Variation of Parameters

Bernd Schröder

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3. The particular solution can be obtained as follows.
   3.1 Assume that the parameters in the solution of the homogeneous equation are functions. (Hence the name.)
   3.2 Substitute the expression into the inhomogeneous equation and solve for the parameters.

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$$y_p(x) = -y_1(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt + y_2(x)\int_{x_0}^{x} \frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt,$$

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where y_1, y_2 are linearly independent solutions of the corresponding homogeneous equation and $W(y_1,y_2) := y_1y_2' - y_2y_1'$.

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$W(y_1,y_2) = \det\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$.

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where y_1, y_2 are linearly independent solutions of the corresponding homogeneous equation and $W(y_1,y_2) := y_1y_2' - y_2y_1'$.
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The Wronskian can also be represented as
$W(y_1,y_2) = \det\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$.
That's it.

Solve the Initial Value Problem $y'' + 4y' - 5y = x$, $y(0) = 1$, $y'(0) = 2$

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$$y''+4y'-5y = 0$$

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Solution of the corresponding homogeneous equation.

$$\begin{aligned} y''+4y'-5y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}-5e^{\lambda x} &= 0 \end{aligned}$$

Solve the Initial Value Problem $y''+4y'-5y=x$, $y(0)=1$, $y'(0)=2$

Solution of the corresponding homogeneous equation.

$$\begin{aligned} y''+4y'-5y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}-5e^{\lambda x} &= 0 \\ \lambda^2+4\lambda-5 &= 0 \end{aligned}$$

Solve the Initial Value Problem $y''+4y'-5y=x$, $y(0)=1$, $y'(0)=2$

Solution of the corresponding homogeneous equation.

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\begin{aligned}
y''+4y'-5y &= 0\\
\lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}-5e^{\lambda x} &= 0\\
\lambda^2+4\lambda-5 &= 0\\
\lambda_{1,2} &= \frac{-4\pm\sqrt{16+20}}{2}
\end{aligned}
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Solve the Initial Value Problem $y''+4y'-5y=x$, $y(0)=1$, $y'(0)=2$

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Solve the Initial Value Problem $y'' + 4y' - 5y = x$, $y(0) = 1$, $y'(0) = 2$

Solution of the corresponding homogeneous equation.

$$\begin{aligned} y'' + 4y' - 5y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} - 5e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda - 5 &= 0 \\ \lambda_{1,2} &= \frac{-4 \pm \sqrt{16+20}}{2} = 1, -5 \\ y_h &= c_1 e^x + c_2 e^{-5x} \end{aligned}$$

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$$W(y_1, y_2)(t) = e^t(-5)e^{-5t} - e^{-5t}e^t$$

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Computing the Wronskian.

$$\begin{aligned} W(y_1, y_2)(t) &= e^t(-5)e^{-5t} - e^{-5t}e^t \\ &= -5e^{-4t} - e^{-4t} \end{aligned}$$

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Computing the Wronskian.

$$\begin{aligned} W(y_1, y_2)(t) &= e^t(-5)e^{-5t} - e^{-5t}e^t \\ &= -5e^{-4t} - e^{-4t} \\ &= -6e^{-4t} \end{aligned}$$

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Computing the integrals.

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Computing the integrals.

$$\int \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_2(t)\, dt$$

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$$\int \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_2(t)\, dt \quad = \quad \int \frac{1}{-6e^{-4t}} \frac{t}{1} e^{-5t}\, dt$$

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&= -\frac{1}{6} \int te^{5t}\, dt \\
&= -\frac{1}{6} \left[t\frac{1}{5}e^{5t} - \int \frac{1}{5}e^{5t}\, dt \right] \\
&= -\frac{1}{30} \left[te^{5t} - \int e^{5t}\, dt \right]
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Stating the general solution.

$$\begin{aligned} y_p &= -y_1(x) \int_{x_0}^{x} \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_2(t)\, dt + y_2(x) \int_{x_0}^{x} \frac{1}{W(y_1, y_2)(t)} \frac{f(t)}{a_2(t)} y_1(t)\, dt \\ &= -e^x \left[-\frac{1}{6} \left[-xe^{-x} - e^{-x} \right] \right] \end{aligned}$$

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Stating the general solution.

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$$\begin{aligned}
y_p &= -y_1(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_2(t)\,dt + y_2(x)\int_{x_0}^{x}\frac{1}{W(y_1,y_2)(t)}\frac{f(t)}{a_2(t)}y_1(t)\,dt \\
&= -e^x\left[-\frac{1}{6}\left[-xe^{-x} - e^{-x}\right]\right] + e^{-5x}\left[-\frac{1}{30}\left[xe^{5x} - \frac{1}{5}e^{5x}\right]\right] \\
&= -\frac{1}{6}x - \frac{1}{6} - \frac{1}{30}x + \frac{1}{150} \\
&= -\frac{1}{5}x - \frac{4}{25}
\end{aligned}$$

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&= -\frac{1}{6}x - \frac{1}{6} - \frac{1}{30}x + \frac{1}{150} \\
&= -\frac{1}{5}x - \frac{4}{25}
\end{aligned}$$

$$y = -\frac{1}{5}x - \frac{4}{25} + c_1e^{x} + c_2e^{-5x}$$

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$$\begin{aligned} y &= -\frac{1}{5}x - \frac{4}{25} + c_1 e^x + c_2 e^{-5x} \\ y' &= -\frac{1}{5} + c_1 e^x - 5c_2 e^{-5x} \end{aligned}$$

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1 &= y(0)
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Solve the Initial Value Problem $y''+4y'-5y=x$, $y(0)=1$, $y'(0)=2$, finding c_1, c_2.

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\begin{aligned}
y &= -\frac{1}{5}x-\frac{4}{25}+c_1e^x+c_2e^{-5x} \\
y' &= -\frac{1}{5}+c_1e^x-5c_2e^{-5x} \\
1 &= y(0)=-\frac{4}{25}+c_1+c_2 \\
2 &= y'(0)
\end{aligned}
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y &= -\frac{1}{5}x-\frac{4}{25}+c_1e^x+c_2e^{-5x} \\
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1 &= y(0)=-\frac{4}{25}+c_1+c_2 \\
2 &= y'(0)=-\frac{1}{5}+c_1-5c_2 \\
-1 &= \frac{1}{25}+6c_2
\end{aligned}$$

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y &= -\frac{1}{5}x - \frac{4}{25} + c_1 e^x + c_2 e^{-5x} \\
y' &= -\frac{1}{5} + c_1 e^x - 5c_2 e^{-5x} \\
1 &= y(0) = -\frac{4}{25} + c_1 + c_2 \\
2 &= y'(0) = -\frac{1}{5} + c_1 - 5c_2 \\
-1 &= \frac{1}{25} + 6c_2, \qquad c_2 = -\frac{13}{75}
\end{aligned}$$

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1 &= y(0) = -\frac{4}{25} + c_1 + c_2 \\
2 &= y'(0) = -\frac{1}{5} + c_1 - 5c_2 \\
-1 &= \frac{1}{25} + 6c_2, \qquad c_2 = -\frac{13}{75}, \qquad c_1 = \frac{29}{25} - c_2
\end{aligned}
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-1 &= \frac{1}{25} + 6c_2, \qquad c_2 = -\frac{13}{75}, \qquad c_1 = \frac{29}{25} - c_2 = \frac{4}{3}
\end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' - 5y = x$, $y(0) = 1$, $y'(0) = 2$, finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{1}{5}x - \frac{4}{25} + c_1 e^x + c_2 e^{-5x} \\
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y &= -\frac{1}{5}x - \frac{4}{25} + \frac{4}{3}e^x - \frac{13}{75}e^{-5x}
\end{aligned}$$

Does $y = -\frac{1}{5}x - \frac{4}{25} + \frac{4}{3}e^x - \frac{13}{75}e^{-5x}$ Really Solve the Initial Value Problem $y'' + 4y' - 5y = x,\ y(0) = 1,\ y'(0) = 2$?

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x &\stackrel{?}{=} x\left[(-5)\left(-\frac{1}{5}\right)\right]
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$$x \stackrel{?}{=} x\left[(-5)\left(-\frac{1}{5}\right)\right] + \left[4\left(-\frac{1}{5}\right) + (-5)\left(-\frac{4}{25}\right)\right] + e^x\left[\frac{4}{3} + 4\cdot\frac{4}{3} + (-5)\frac{4}{3}\right]$$

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&\quad +e^{-5x}\left[-\frac{13}{3}+4\cdot\frac{13}{15}+(-5)\left(-\frac{13}{75}\right)\right]=x \quad \surd
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6. We now have a tool that can handle real life situations.