Diagonalizable Homogeneous Systems of Linear Differential Equations with Constant Coefficients

Bernd Schröder

Homogeneous Systems of Linear Differential Equations with Constant Coefficients

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$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}\,' = (\Phi\vec{x})',$$

that is, $\vec{y} = \Phi\vec{x}$ solves $\vec{y}\,' = A\vec{y}$.

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4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}' = (\Phi\vec{x})',$$

   that is, $\vec{y} = \Phi\vec{x}$ solves $\vec{y}' = A\vec{y}$.
5. Conversely, every solution of $\vec{y}' = A\vec{y}$ can be obtained as above.

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6. So if we can find a representation $A = \Phi D \Phi^{-1}$ so that $\vec{x}' = D\vec{x}$ is easy to solve, then $\vec{y}' = A\vec{y}$ is also easy to solve.

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7. An $n \times n$ matrix A is called **diagonalizable** if and only if there are a diagonal matrix $D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$, and an invertible matrix Φ so that $A = \Phi D \Phi^{-1}$.

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8. If D is a diagonal matrix, then the solutions of $\vec{x}' = D\vec{x}$ are $e^{\lambda_1 t}\vec{e}_1, \ldots, e^{\lambda_n t}\vec{e}_n$, because the individual equations are of the form $x_j' = \lambda_j x_j$.

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9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.

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10. The columns of Φ satisfy

$$A\Phi_j$$

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11. Eigenvalues can be computed by solving the equation $\det(A - \lambda I) = 0$, where I is the identity matrix.
12. Corresponding eigenvectors are computed with systems of equations $A\vec{v} = \lambda_j \vec{v}$ or, more commonly $(A - \lambda_j I)\vec{v} = \vec{0}$.

Solve the System $\vec{y}' = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 3 & -4 \\ 1 & 1 & 0 \end{pmatrix} \vec{y}$

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$$\begin{aligned} &= (-\lambda)\det\begin{pmatrix} 3-\lambda & -4 \\ 1 & -\lambda \end{pmatrix} - 5\det\begin{pmatrix} 2 & -2 \\ 1 & -\lambda \end{pmatrix} \\ &\quad + 1\cdot\det\begin{pmatrix} 2 & -2 \\ 3-\lambda & -4 \end{pmatrix} \\ &= (-\lambda)\Big((3-\lambda)(-\lambda)-(-4)\cdot 1\Big) \end{aligned}$$

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&= (-\lambda)\big((3-\lambda)(-\lambda)-(-4)\cdot 1\big) - 5\big(2\cdot(-\lambda)-(-2)\cdot 1\big) \\
&\quad + \big(2\cdot(-4)-(-2)(3-\lambda)\big) \\
&= (-\lambda)\big(-3\lambda+\lambda^2+4\big)
\end{aligned}$$

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\end{aligned}$$

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$$= (-\lambda)\left(-3\lambda+\lambda^2+4\right) - 5\left(-2\lambda+2\right) + \left(-2-2\lambda\right)$$

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&= (-\lambda)\big(-3\lambda+\lambda^2+4\big) - 5\big(-2\lambda+2\big) + \big(-2-2\lambda\big) \\
&= -\lambda^3
\end{aligned}$$

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&= (-\lambda)\big(-3\lambda+\lambda^2+4\big) - 5\big(-2\lambda+2\big) + \big(-2-2\lambda\big) \\
&= -\lambda^3+3\lambda^2
\end{aligned}$$

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&= -\lambda^3+3\lambda^2+4\lambda-12
\end{aligned}$$

Zeros of $-\lambda^3+3\lambda^2+4\lambda-12$

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Rational Zeros Theorem

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$p=\pm 1,\pm 2,\pm 3,\pm 4,\pm 6,\pm 12$

Zeros of $-\lambda^3 + 3\lambda^2 + 4\lambda - 12$

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$p = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$q = \pm1.$

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$$
\begin{array}{r|rrrr}
2 & -1 & 3 & 4 & -12 \\
 & & -2 & 2 & 12 \\
\hline
 & -1 & 1 & 6 & \boxed{0}
\end{array}
$$

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$$\begin{array}{c|rrrr} 2 & -1 & 3 & 4 & -12 \\ & & -2 & 2 & 12 \\ \hline & -1 & 1 & 6 & \boxed{0} \end{array}$$

$$-\lambda^2+\lambda+6 \quad = \quad 0$$

Zeros of $-\lambda^3+3\lambda^2+4\lambda-12$

Rational Zeros Theorem

$p=\pm1,\pm2,\pm3,\pm4,\pm6,\pm12$

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$$\begin{aligned} -\lambda^2+\lambda+6 &= 0 \\ \lambda_{1,2} &= \frac{-1\pm\sqrt{1+24}}{-2} \end{aligned}$$

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$$-\lambda^3+3\lambda^2+4\lambda-12=-(\lambda-2)$$

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$$
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2 & -1 & 3 & 4 & -12 \\
 & & -2 & 2 & 12 \\
\hline
 & -1 & 1 & 6 & \boxed{0}
\end{array}
$$

$$
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\lambda_{1,2} &= \frac{-1\pm\sqrt{1+24}}{-2}=\frac{-1\pm 5}{-2} \\
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$$

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Eigenvector for $\lambda = 2$

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$$\begin{pmatrix} 0-\lambda & 2 & -2 \\ 5 & 3-\lambda & -4 \\ 1 & 1 & -\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 2v_3 & = & 0 \\ & & 4v_2 & - & 6v_3 & = & 0 \\ & - & 4v_2 & + & 6v_3 & = & 0 \end{array}$$

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$v_3 = 2$

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$$v_3 = 2,\ v_2 = \frac{3}{2}v_3 = 3$$

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-2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\
5v_1 & + & v_2 & - & 4v_3 & = & 0
\end{array}
$$

$$
\begin{array}{rcrcrcl}
v_1 & + & v_2 & - & 2v_3 & = & 0 \\
 & & 4v_2 & - & 6v_3 & = & 0 \\
 & - & 4v_2 & + & 6v_3 & = & 0
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$$v_3 = 2,\ v_2 = \frac{3}{2}v_3 = 3,\ v_1 = -v_2 + 2v_3 = -3 + 2 \cdot 2$$

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Eigenvector for $\lambda = -2$

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Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

Eigenvector for $\lambda = -2$

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v_1 & + & v_2 & + & 2v_3 & = & 0
\end{array}$$

$$\begin{array}{rcrcrcl}
v_1 & + & v_2 & + & 2v_3 & = & 0 \\
 & & & - & 6v_3 & = & 0 \\
 & & & - & 14v_3 & = & 0
\end{array}$$

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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$v_3 = 0$

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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$v_3 = 0$ (not a choice)

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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$v_3 = 0$ (not a choice), $v_2 = -v_1$

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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$v_3 = 0$ (not a choice), $v_2 = -v_1 := -1$

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

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Eigenvector for $\lambda = -2$

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Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & + & 2v_3 & = & 0 \\ & & & - & 6v_3 & = & 0 \\ & & & - & 14v_3 & = & 0 \end{array}$$

$v_3 = 0$ (not a choice), $v_2 = -v_1 := -1$ (chosen),

$$\vec{v}_{-2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \text{check: } \begin{pmatrix} 0 & 2 & -2 \\ 5 & 3 & -4 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = -2$

$$\begin{array}{rcrcrcl} 2v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & + & 5v_2 & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & + & 2v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & + & 2v_3 & = & 0 \\ & & & - & 6v_3 & = & 0 \\ & & & - & 14v_3 & = & 0 \end{array}$$

$v_3 = 0$ (not a choice), $v_2 = -v_1 := -1$ (chosen),

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Eigenvector for $\lambda = 3$

Eigenvector for $\lambda = 3$

$$\begin{pmatrix} 0-\lambda & 2 & -2 \\ 5 & 3-\lambda & -4 \\ 1 & 1 & -\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = 3$

$$\begin{pmatrix} 0-\lambda & 2 & -2 \\ 5 & 3-\lambda & -4 \\ 1 & 1 & -\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 & -2 \\ 5 & 0 & -4 \\ 1 & 1 & -3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$v_3 := 5$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$v_3 := 5,\ v_2 = \frac{11}{5} v_3$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11,\ v_1 = -v_2 + 3v_3$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

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$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11,\ v_1 = -v_2 + 3v_3 = -11 + 3 \cdot 5$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11,\ v_1 = -v_2 + 3v_3 = -11 + 3 \cdot 5 = 4$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11,\ v_1 = -v_2 + 3v_3 = -11 + 3 \cdot 5 = 4$$

$$\vec{v}_3 = \begin{pmatrix} 4 \\ 11 \\ 5 \end{pmatrix}$$

Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

$$\begin{array}{rcrcrcl} v_1 & + & v_2 & - & 3v_3 & = & 0 \\ & + & 5v_2 & - & 11v_3 & = & 0 \\ & - & 5v_2 & + & 11v_3 & = & 0 \end{array}$$

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$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

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$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

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Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

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$$v_3 := 5,\ v_2 = \frac{11}{5}v_3 = 11,\ v_1 = -v_2 + 3v_3 = -11 + 3 \cdot 5 = 4$$

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Eigenvector for $\lambda = 3$

$$\begin{array}{rcrcrcl} -3v_1 & + & 2v_2 & - & 2v_3 & = & 0 \\ 5v_1 & & & - & 4v_3 & = & 0 \\ v_1 & + & v_2 & - & 3v_3 & = & 0 \end{array}$$

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General Solution of the System

$$\vec{y}' = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 3 & -4 \\ 1 & 1 & 0 \end{pmatrix} \vec{y}$$

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$$\vec{y}' = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 3 & -4 \\ 1 & 1 & 0 \end{pmatrix} \vec{y}$$

$$\vec{y} = c_1 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} e^{2t}$$

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General Solution of the System

$$\vec{y}' = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 3 & -4 \\ 1 & 1 & 0 \end{pmatrix} \vec{y}$$

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Solve the System $\vec{y}\,' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

Solve the System $\vec{y}\,' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix}$$

Solve the System $\vec{y}\,' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

$$\det\begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix}$$

$$= \quad (-\lambda)\det\begin{pmatrix} -2-\lambda & 0 \\ -2 & -4-\lambda \end{pmatrix}$$

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$$= \quad (-\lambda)\det \begin{pmatrix} -2-\lambda & 0 \\ -2 & -4-\lambda \end{pmatrix} - 1\cdot\det \begin{pmatrix} 8 & 0 \\ -2 & -4-\lambda \end{pmatrix}$$

Solve the System $\vec{y}' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

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$$= (-\lambda)\det\begin{pmatrix} -2-\lambda & 0 \\ -2 & -4-\lambda \end{pmatrix} - 1\cdot\det\begin{pmatrix} 8 & 0 \\ -2 & -4-\lambda \end{pmatrix}$$

$$+(-1)\det\begin{pmatrix} 8 & 0 \\ -2-\lambda & 0 \end{pmatrix}$$

Solve the System $\vec{y}' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix}$$

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$$+(-1)\det\begin{pmatrix} 8 & 0 \\ -2-\lambda & 0 \end{pmatrix}$$

$$= (-\lambda)(-2-\lambda)(-4-\lambda) - 8(-4-\lambda) + (-1)\cdot 0$$

Solve the System $\vec{y}' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix}$$

$$\begin{aligned}
&= (-\lambda)\det\begin{pmatrix} -2-\lambda & 0 \\ -2 & -4-\lambda \end{pmatrix} - 1\cdot\det\begin{pmatrix} 8 & 0 \\ -2 & -4-\lambda \end{pmatrix} \\
&\quad +(-1)\det\begin{pmatrix} 8 & 0 \\ -2-\lambda & 0 \end{pmatrix} \\
&= (-\lambda)(-2-\lambda)(-4-\lambda) - 8(-4-\lambda) + (-1)\cdot 0 \\
&= -\lambda^3 - 6\lambda^2 - 8\lambda + 8\lambda + 32
\end{aligned}$$

Solve the System $\vec{y}' = \begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \vec{y}$

$$
\begin{aligned}
& \det \begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix} \\
= \; & (-\lambda)\det \begin{pmatrix} -2-\lambda & 0 \\ -2 & -4-\lambda \end{pmatrix} - 1 \cdot \det \begin{pmatrix} 8 & 0 \\ -2 & -4-\lambda \end{pmatrix} \\
& +(-1)\det \begin{pmatrix} 8 & 0 \\ -2-\lambda & 0 \end{pmatrix} \\
= \; & (-\lambda)(-2-\lambda)(-4-\lambda) - 8(-4-\lambda) + (-1)\cdot 0 \\
= \; & -\lambda^3 - 6\lambda^2 - 8\lambda + 8\lambda + 32 \\
= \; & -(\lambda-2)(\lambda+4)^2
\end{aligned}
$$

Eigenvector for $\lambda = 2$

Eigenvector for $\lambda = 2$

$$\begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{pmatrix} 0-\lambda & 8 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & -2 & -4-\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 8 & 0 \\ 1 & -4 & 0 \\ -1 & -2 & -6 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

$$v_3 = \frac{-v_1 - 2v_2}{6}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

$$v_3 = \frac{-v_1 - 2v_2}{6} = \frac{-4 - 2\cdot 1}{6}$$

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$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1,\ v_1 = 4v_2 = 4,$

$$v_3 = \frac{-v_1 - 2v_2}{6} = \frac{-4 - 2 \cdot 1}{6} = -\frac{6}{6}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

$$v_3 = \frac{-v_1 - 2v_2}{6} = \frac{-4 - 2 \cdot 1}{6} = -\frac{6}{6} = -1$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1,\ v_1 = 4v_2 = 4,$

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$$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

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$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, check:

$$\begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

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$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, check:

$$\begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

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$v_2 := 1$, $v_1 = 4v_2 = 4$,

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$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, check:

$$\begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ -2 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcl} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

$$v_3 = \frac{-v_1 - 2v_2}{6} = \frac{-4 - 2 \cdot 1}{6} = -\frac{6}{6} = -1$$

$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, check:

$$\begin{pmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

Eigenvector for $\lambda = 2$

$$\begin{array}{rcrcrcr} -2v_1 & + & 8v_2 & & & = & 0 \\ v_1 & - & 4v_2 & & & = & 0 \\ -1v_1 & - & 2v_2 & - & 6v_3 & = & 0 \end{array}$$

$v_2 := 1$, $v_1 = 4v_2 = 4$,

$$v_3 = \frac{-v_1 - 2v_2}{6} = \frac{-4 - 2\cdot 1}{6} = -\frac{6}{6} = -1$$

$\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, check:

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6. But the fundamental ideas were discussed here.