Homogeneous Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

Bernd Schröder

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}\,' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x})$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x})$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x}$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}'$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}' = (\Phi\vec{x})',$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}' = (\Phi\vec{x})',$$

that is, $\vec{y} = \Phi\vec{x}$ solves $\vec{y}' = A\vec{y}$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

1. These systems are typically written in matrix form as $\vec{y}' = A\vec{y}$, where A is an $n \times n$ matrix and $\vec{y}$ is a column vector with n rows.
2. The theory guarantees that there will always be a set of n linearly independent solutions $\{\vec{y}_1, \ldots, \vec{y}_n\}$.
3. Every solution is of the form $\vec{y} = c_1\vec{y}_1 + \cdots + c_n\vec{y}_n$.
4. If $A = \Phi D \Phi^{-1}$ and $\vec{x}$ solves $\vec{x}' = D\vec{x}$, then

$$A(\Phi\vec{x}) = \Phi D \Phi^{-1}(\Phi\vec{x}) = \Phi D\vec{x} = \Phi\vec{x}' = (\Phi\vec{x})',$$

   that is, $\vec{y} = \Phi\vec{x}$ solves $\vec{y}' = A\vec{y}$.
5. Conversely, every solution of $\vec{y}' = A\vec{y}$ can be obtained as above.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

6. So if we can find a representation $A = \Phi D \Phi^{-1}$ so that $\vec{x}' = D\vec{x}$ is easy to solve, then $\vec{y}' = A\vec{y}$ is also easy to solve.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

6. So if we can find a representation $A = \Phi D \Phi^{-1}$ so that $\vec{x}' = D\vec{x}$ is easy to solve, then $\vec{y}' = A\vec{y}$ is also easy to solve.
7. An $n \times n$ matrix A is called **diagonalizable** if and only if there are a diagonal matrix $D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$, and an invertible matrix Φ so that $A = \Phi D \Phi^{-1}$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

6. So if we can find a representation $A = \Phi D \Phi^{-1}$ so that $\vec{x}' = D\vec{x}$ is easy to solve, then $\vec{y}' = A\vec{y}$ is also easy to solve.
7. An $n \times n$ matrix A is called **diagonalizable** if and only if there are a diagonal matrix $D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$, and an invertible matrix Φ so that $A = \Phi D \Phi^{-1}$.
8. If D is a diagonal matrix, then the solutions of $\vec{x}' = D\vec{x}$ are $e^{\lambda_1 t}\vec{e}_1, \ldots, e^{\lambda_n t}\vec{e}_n$, because the individual equations are of the form $x_j' = \lambda_j x_j$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

6. So if we can find a representation $A = \Phi D \Phi^{-1}$ so that $\vec{x}' = D\vec{x}$ is easy to solve, then $\vec{y}' = A\vec{y}$ is also easy to solve.
7. An $n \times n$ matrix A is called **diagonalizable** if and only if there are a diagonal matrix $D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$, and an invertible matrix Φ so that $A = \Phi D \Phi^{-1}$.
8. If D is a diagonal matrix, then the solutions of $\vec{x}' = D\vec{x}$ are $e^{\lambda_1 t}\vec{e}_1, \ldots, e^{\lambda_n t}\vec{e}_n$, because the individual equations are of the form $x_j' = \lambda_j x_j$. (λ_j can be real or complex.)

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A=\Phi D\Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}'=A\vec{y}$ is $\vec{y}=e^{\lambda_1 t}\Phi_1+\cdots+e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1} \Phi_j$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1}\Phi_j = \Phi D \vec{e}_j$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A=\Phi D\Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}'=A\vec{y}$ is $\vec{y}=e^{\lambda_1 t}\Phi_1+\cdots+e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j=\Phi D\Phi^{-1}\Phi_j=\Phi D\vec{e}_j=\Phi\lambda_j\vec{e}_j$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1}\Phi_j = \Phi D\vec{e}_j = \Phi\lambda_j\vec{e}_j = \lambda_j\Phi_j.$$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1}\Phi_j = \Phi D \vec{e}_j = \Phi \lambda_j \vec{e}_j = \lambda_j \Phi_j.$$

Nonzero vectors with this property are called **eigenvectors**. λ_j is called an **eigenvalue**.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1}\Phi_j = \Phi D \vec{e}_j = \Phi \lambda_j \vec{e}_j = \lambda_j \Phi_j.$$

Nonzero vectors with this property are called **eigenvectors**. λ_j is called an **eigenvalue**.

11. Eigenvalues can be computed by solving the equation $\det(A - \lambda I) = 0$, where I is the identity matrix.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A = \Phi D \Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}' = A\vec{y}$ is $\vec{y} = e^{\lambda_1 t}\Phi_1 + \cdots + e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy

$$A\Phi_j = \Phi D \Phi^{-1}\Phi_j = \Phi D \vec{e}_j = \Phi \lambda_j \vec{e}_j = \lambda_j \Phi_j.$$

    Nonzero vectors with this property are called **eigenvectors**. λ_j is called an **eigenvalue**.
11. Eigenvalues can be computed by solving the equation $\det(A - \lambda I) = 0$, where I is the identity matrix.
12. Corresponding eigenvectors are computed with systems of equations $A\vec{v} = \lambda_j \vec{v}$

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

9. That means, if $A=\Phi D\Phi^{-1}$ and D is a diagonal matrix, then the solution of $\vec{y}'=A\vec{y}$ is $\vec{y}=e^{\lambda_1 t}\Phi_1+\cdots+e^{\lambda_n t}\Phi_n$, where Φ_j denotes the j^{th} column of the matrix Φ.
10. The columns of Φ satisfy
$$A\Phi_j=\Phi D\Phi^{-1}\Phi_j=\Phi D\vec{e}_j=\Phi\lambda_j\vec{e}_j=\lambda_j\Phi_j.$$
Nonzero vectors with this property are called **eigenvectors**. λ_j is called an **eigenvalue**.
11. Eigenvalues can be computed by solving the equation $\det(A-\lambda I)=0$, where I is the identity matrix.
12. Corresponding eigenvectors are computed with systems of equations $A\vec{v}=\lambda_j\vec{v}$ or, more commonly $(A-\lambda_j I)\vec{v}=\vec{0}$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

13. But if an eigenvalue is complex, we might still want to have a real-valued solution.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

13. But if an eigenvalue is complex, we might still want to have a real-valued solution.
14. Let $\vec{v}$ be an eigenvector of A for the eigenvalue $\lambda = \alpha + i\beta$ with $\beta \neq 0$.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

13. But if an eigenvalue is complex, we might still want to have a real-valued solution.
14. Let $\vec{v}$ be an eigenvector of A for the eigenvalue $\lambda = \alpha + i\beta$ with $\beta \neq 0$. Then $\overline{\vec{v}}$ is an eigenvector of A for the eigenvalue $\overline{\lambda}$, where $\overline{\vec{v}}$ is obtained from $\vec{v}$ by replacing every component with its complex conjugate.

Systems of Linear Differential Equations with Constant Coefficients and Complex Eigenvalues

13. But if an eigenvalue is complex, we might still want to have a real-valued solution.
14. Let $\vec{v}$ be an eigenvector of A for the eigenvalue $\lambda = \alpha + i\beta$ with $\beta \neq 0$. Then $\overline{\vec{v}}$ is an eigenvector of A for the eigenvalue $\overline{\lambda}$, where $\overline{\vec{v}}$ is obtained from $\vec{v}$ by replacing every component with its complex conjugate.
15. The functions $\left(\Re(\vec{v})e^{\alpha t}\cos(\beta t) - \Im(\vec{v})e^{\alpha t}\sin(\beta t)\right)$ and $\left(\Im(\vec{v})e^{\alpha t}\cos(\beta t) + \Re(\vec{v})e^{\alpha t}\sin(\beta t)\right)$ are two linearly independent real solutions of the system of linear differential equations $\vec{y}' = A\vec{y}$. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of the vector, taken componentwise.

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix}$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} = (3-\lambda)(3-\lambda)$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\det \begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} = (3-\lambda)(3-\lambda) - 1\cdot(-4)$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned} \det\begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} &= (3-\lambda)(3-\lambda) - 1\cdot(-4) \\ &= 9 - 6\lambda + \lambda^2 + 4 \end{aligned}$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned} \det\begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} &= (3-\lambda)(3-\lambda) - 1\cdot(-4) \\ &= 9 - 6\lambda + \lambda^2 + 4 \\ &= \lambda^2 - 6\lambda + 13 \end{aligned}$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned}
\det\begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} &= (3-\lambda)(3-\lambda) - 1\cdot(-4) \\
&= 9 - 6\lambda + \lambda^2 + 4 \\
&= \lambda^2 - 6\lambda + 13 \\
\lambda_{1,2} &= \frac{-(-6) \pm \sqrt{36 - 4\cdot 13}}{2}
\end{aligned}$$

Solve the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned} \det\begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} &= (3-\lambda)(3-\lambda) - 1\cdot(-4) \\ &= 9 - 6\lambda + \lambda^2 + 4 \\ &= \lambda^2 - 6\lambda + 13 \\ \lambda_{1,2} &= \frac{-(-6) \pm \sqrt{36 - 4\cdot 13}}{2} = \frac{6 \pm 4i}{2} \end{aligned}$$

Solve the System $\vec{y}\,' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\det\begin{pmatrix} 3-\lambda & 1 \\ -4 & 3-\lambda \end{pmatrix} = (3-\lambda)(3-\lambda) - 1\cdot(-4)$$

$$= 9 - 6\lambda + \lambda^2 + 4$$

$$= \lambda^2 - 6\lambda + 13$$

$$\lambda_{1,2} = \frac{-(-6) \pm \sqrt{36 - 4\cdot 13}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$$v_2 = 2iv_1,\ v_1 := 1,\ v_2 = 2i,\ \vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}.$$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$, $\vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$.

Check:

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$, $\vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$.

Check: $\begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2i \end{pmatrix}$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$, $\vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$.

Check: $\begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2i \end{pmatrix} = \begin{pmatrix} 3+2i \\ -4+6i \end{pmatrix}$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$, $\vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$.

Check: $\begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2i \end{pmatrix} = \begin{pmatrix} 3+2i \\ -4+6i \end{pmatrix} = (3+2i) \begin{pmatrix} 1 \\ 2i \end{pmatrix}$

Eigenvector for $\lambda = 3+2i$

$$\begin{pmatrix} 3-(3+2i) & 1 \\ -4 & 3-(3+2i) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{rcrcl} -2iv_1 & + & 1v_2 & = & 0 \\ -4v_1 & - & 2iv_2 & = & 0 \end{array}$$

$v_2 = 2iv_1$, $v_1 := 1$, $v_2 = 2i$, $\vec{v} = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$.

Check: $\begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2i \end{pmatrix} = \begin{pmatrix} 3+2i \\ -4+6i \end{pmatrix} = (3+2i) \begin{pmatrix} 1 \\ 2i \end{pmatrix} \quad \surd$

General Solution of the System $\vec{y}\,' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

General Solution of the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\vec{y} \quad =$$

General Solution of the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\vec{y} = c_1 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{3t} \cos(2t) \right.$$

General Solution of the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\vec{y} = c_1 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{3t} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t} \sin(2t) \right]$$

General Solution of the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned} \vec{y} &= c_1 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{3t} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t} \sin(2t) \right] \\ &\quad + c_2 \left[\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t} \cos(2t) \right. \end{aligned}$$

General Solution of the System $\vec{y}' = \begin{pmatrix} 3 & 1 \\ -4 & 3 \end{pmatrix} \vec{y}$

$$\begin{aligned} \vec{y} &= c_1 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{3t} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t} \sin(2t) \right] \\ &\quad + c_2 \left[\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t} \cos(2t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{3t} \sin(2t) \right] \end{aligned}$$