Finite Difference Method

Bernd Schröder

Start with the Grid

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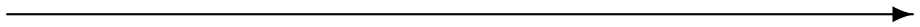

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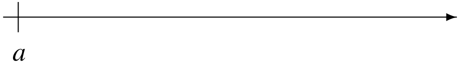

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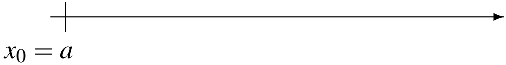

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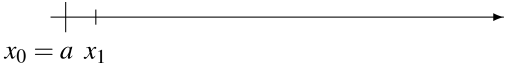

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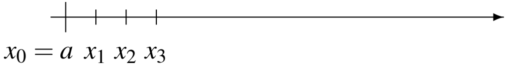

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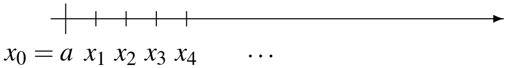

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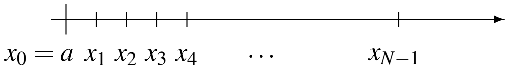

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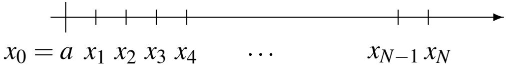

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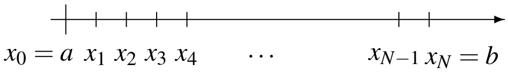

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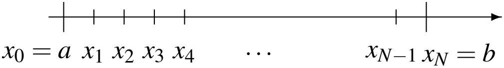

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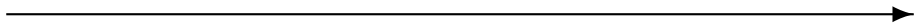

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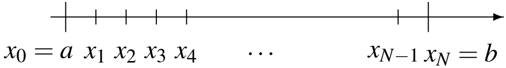

Find Approximate Expressions for the Derivatives

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   replace x_n with $x_0 + nh$,
   and replace h with its numerical value.
5. Solve the resulting system of equations.

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y(x+h) - 2y(x) + y(x-h) - xh\big(y(x+h) - y(x-h)\big) - 2h^2 y(x) &= 0\\
y(x+h)\,(1 - xh) + y(x)\left(-2 - 2h^2\right) + y(x-h)\,(1 + xh) &= 0\\
y(x_n+h)\,(1 - x_n h) + y(x_n)\left(-2 - 2h^2\right) + y(x_n-h)\,(1 + x_n h) &= 0\\
y_{n+1}\left(1 - \frac{1}{64}n\right) + y_n\left(-2 - \frac{1}{32}\right) + y_{n-1}\left(1 + \frac{1}{64}n\right) &= 0
\end{aligned}$$

Use Finite Differences with 8 Intervals to Solve the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

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Boundaries :

Use Finite Differences with 8 Intervals to Solve the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

$$\text{Boundaries}: y_{n+1}\left(1 - \frac{1}{64}n\right) + y_n\left(-2 - \frac{1}{32}\right) + y_{n-1}\left(1 + \frac{1}{64}n\right) \quad = \quad 0$$

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$$n = 1: \quad y_2\frac{63}{64} + y_1\left(-\frac{65}{32}\right) + y_0\frac{65}{64} = 0$$

Use Finite Differences with 8 Intervals to Solve the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

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\text{Boundaries}: y_{n+1}\left(1-\frac{1}{64}n\right)+y_n\left(-2-\frac{1}{32}\right)+y_{n-1}\left(1+\frac{1}{64}n\right) &= 0\\
n=1: \quad y_2\frac{63}{64}+y_1\left(-\frac{65}{32}\right)+y_0\frac{65}{64} &= 0\\
y_2\frac{63}{64}+y_1\left(-\frac{65}{32}\right) &= -\frac{65}{64}
\end{aligned}$$

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$$
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\text{Boundaries}: y_{n+1}\left(1-\frac{1}{64}n\right)+y_n\left(-2-\frac{1}{32}\right)+y_{n-1}\left(1+\frac{1}{64}n\right) &= 0\\
n=1: \quad y_2\frac{63}{64}+y_1\left(-\frac{65}{32}\right)+y_0\frac{65}{64} &= 0\\
y_2\frac{63}{64}+y_1\left(-\frac{65}{32}\right) &= -\frac{65}{64}\\
n=7: \quad y_8\frac{57}{64}+y_7\left(-\frac{65}{32}\right)+y_6\frac{71}{64} &= 0
\end{aligned}
$$

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y_2\frac{63}{64}+y_1\left(-\frac{65}{32}\right) &= -\frac{65}{64}\\
n=7: \quad y_8\frac{57}{64}+y_7\left(-\frac{65}{32}\right)+y_6\frac{71}{64} &= 0\\
y_7\left(-\frac{65}{32}\right)+y_6\frac{71}{64} &= -e\frac{57}{64}
\end{aligned}
$$

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Use Finite Differences with 8 Intervals to Solve the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

$$A = \begin{pmatrix} -2.0313 & 0.9844 & 0 & 0 & 0 & 0 & 0 \\ 1.0313 & -2.0313 & 0.9688 & 0 & 0 & 0 & 0 \\ 0 & 1.0469 & -2.0313 & 0.9531 & 0 & 0 & 0 \\ 0 & 0 & 1.0625 & -2.0313 & 0.9375 & 0 & 0 \\ 0 & 0 & 0 & 1.0781 & -2.0313 & 0.9219 & 0 \\ 0 & 0 & 0 & 0 & 1.0938 & -2.0313 & 0.9063 \\ 0 & 0 & 0 & 0 & 0 & 1.1094 & -2.0313 \end{pmatrix}$$

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Use Finite Differences with 8 Intervals to Solve the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

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$$y = \begin{pmatrix} 1.0165 \\ 1.0658 \\ 1.1527 \\ 1.2859 \\ 1.4797 \\ 1.7565 \\ 2.1512 \end{pmatrix}$$

Actual Solution of the Boundary Value Problem

$y'' - 2xy' - 2y = 0,\ y(0) = 1,\ y(1) = e$

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$$y'(x) = 2xe^{x^2}$$

Actual Solution of the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

$$
\begin{aligned}
y(x) &= e^{x^2} \\
y(0) = 1 \quad \surd \qquad & \qquad y(1) = e \quad \surd \\
y'(x) &= 2xe^{x^2} \\
y''(x) &= 2e^{x^2} + 4x^2 e^{x^2}
\end{aligned}
$$

Actual Solution of the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

$$\begin{aligned} y(x) &= e^{x^2} \\ y(0)=1 \quad \surd \qquad & \quad y(1) = e \quad \surd \\ y'(x) &= 2xe^{x^2} \\ y''(x) &= 2e^{x^2} + 4x^2 e^{x^2} \\ &= 2y + 2xy' \end{aligned}$$

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&= 2y + 2xy' \quad \surd
\end{aligned}
$$

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$$ys(x) := e^{x^2}$$

Actual Solution of the Boundary Value Problem $y'' - 2xy' - 2y = 0$, $y(0) = 1$, $y(1) = e$

$$ys(x) := e^{x^2}$$

$$s := \begin{pmatrix} ys(x_1) \\ ys(x_2) \\ ys(x_3) \\ ys(x_4) \\ ys(x_5) \\ ys(x_6) \\ ys(x_7) \end{pmatrix}$$

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Actual Solution of the Boundary Value Problem

$$y'' - 2xy' - 2y = 0,\ y(0) = 1,\ y(1) = e$$

$$s = \begin{pmatrix} 1.0157 \\ 1.0645 \\ 1.151 \\ 1.284 \\ 1.4779 \\ 1.7551 \\ 2.1503 \end{pmatrix}$$

Actual Solution of the Boundary Value Problem

$y'' - 2xy' - 2y = 0,\ y(0) = 1,\ y(1) = e$

$$s = \begin{pmatrix} 1.0157 \\ 1.0645 \\ 1.151 \\ 1.284 \\ 1.4779 \\ 1.7551 \\ 2.1503 \end{pmatrix} \qquad y = \begin{pmatrix} 1.0165 \\ 1.0658 \\ 1.1527 \\ 1.2859 \\ 1.4797 \\ 1.7565 \\ 2.1512 \end{pmatrix}$$

The Finite Difference Method

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2. The solution is typically not known.
3. One way to get an indication that we are close to a solution is to refine the grid and compare consecutive approximations. If they are not too far apart, we may be close to the solution.
4. Finite difference methods lead to large systems of linear equations which need to be solved with numerical techniques.