Linear First Order Differential Equations

Bernd Schröder

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That's it.

Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)} y = 1$, $y(1) = 0$.

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$$\mu(x)\left(y' + \frac{\cos(x)}{\sin(x)} y\right) = 1 \cdot \mu(x)$$

Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$.

$$\sin(x)\left(y' + \frac{\cos(x)}{\sin(x)}y\right) = 1 \cdot \sin(x)$$

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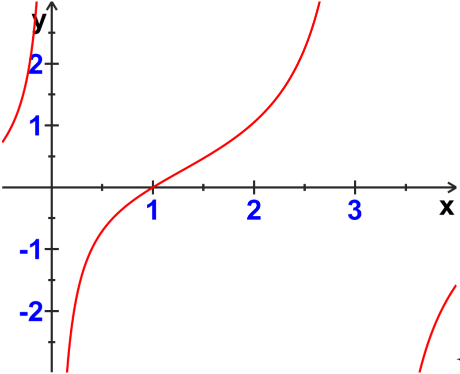

$$y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)}$$

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\frac{1 - \cos^2(x)}{\sin^2(x)} &\stackrel{?}{=} 1 \\
\frac{\sin^2(x)}{\sin^2(x)} &= 1 \ \checkmark
\end{aligned}$$

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Yes, it does