Linear Homogeneous Constant Coefficient Differential Equations

Bernd Schröder

Equation Type and Solution Method

Equation Type and Solution Method

We will focus on linear homogeneous constant coefficient differential equations of second order, because they are encountered most frequently.

1. The general form is $ay'' + by' + cy = 0$, where a, b, c are real constants.

Equation Type and Solution Method

We will focus on linear homogeneous constant coefficient differential equations of second order, because they are encountered most frequently.

1. The general form is $ay'' + by' + cy = 0$, where a, b, c are real constants.
2. Substituting $y = e^{\lambda x}$ into the equation leads to the equation $a\lambda^2 + b\lambda + c = 0$ for λ.

Equation Type and Solution Method

We will focus on linear homogeneous constant coefficient differential equations of second order, because they are encountered most frequently.

1. The general form is $ay'' + by' + cy = 0$, where a, b, c are real constants.
2. Substituting $y = e^{\lambda x}$ into the equation leads to the equation $a\lambda^2 + b\lambda + c = 0$ for λ.
   2.1 If the equation has real solutions $\lambda_1 \neq \lambda_2$, then the general solution is $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$.

Equation Type and Solution Method

We will focus on linear homogeneous constant coefficient differential equations of second order, because they are encountered most frequently.

1. The general form is $ay'' + by' + cy = 0$, where a, b, c are real constants.
2. Substituting $y = e^{\lambda x}$ into the equation leads to the equation $a\lambda^2 + b\lambda + c = 0$ for λ.
   2.1 If the equation has real solutions $\lambda_1 \neq \lambda_2$, then the general solution is $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$.
   2.2 If the equation has complex solutions $\lambda_{1,2} = u \pm iv$, then the general solution is $y = c_1 e^{ux}\cos(vx) + c_2 e^{ux}\sin(vx)$.

Equation Type and Solution Method

We will focus on linear homogeneous constant coefficient differential equations of second order, because they are encountered most frequently.

1. The general form is $ay'' + by' + cy = 0$, where a, b, c are real constants.
2. Substituting $y = e^{\lambda x}$ into the equation leads to the equation $a\lambda^2 + b\lambda + c = 0$ for λ.
   2.1 If the equation has real solutions $\lambda_1 \neq \lambda_2$, then the general solution is $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$.
   2.2 If the equation has complex solutions $\lambda_{1,2} = u \pm iv$, then the general solution is $y = c_1 e^{ux}\cos(vx) + c_2 e^{ux}\sin(vx)$.
   2.3 If the equation has only one real solution λ, then the general solution is $y = c_1 e^{\lambda x} + c_2 x e^{\lambda x}$.

Find the general solution of $2y'' + 5y' + 2y = 0$.

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$2y'' + 5y' + 2y = 0$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$2y'' + 5y' + 2y = 0 \qquad y = e^{\lambda x}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$\begin{aligned} 2y'' + 5y' + 2y &= 0 \qquad y = e^{\lambda x} \\ 2\left(e^{\lambda x}\right)'' + 5\left(e^{\lambda x}\right)' + 2\left(e^{\lambda x}\right) &= 0 \end{aligned}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$\begin{aligned} 2y'' + 5y' + 2y &= 0 \\ 2\left(e^{\lambda x}\right)'' + 5\left(e^{\lambda x}\right)' + 2e^{\lambda x} &= 0 \end{aligned} \qquad y = e^{\lambda x}$$

Find the general solution of $2y''+5y'+2y=0$.

$$\begin{aligned} 2y''+5y'+2y &= 0 \\ 2\left(e^{\lambda x}\right)''+5\lambda e^{\lambda x}+2e^{\lambda x} &= 0 \end{aligned} \qquad y=e^{\lambda x}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$\begin{aligned} 2y'' + 5y' + 2y &= 0 \qquad\qquad y = e^{\lambda x} \\ 2\lambda^2 e^{\lambda x} + 5\lambda e^{\lambda x} + 2e^{\lambda x} &= 0 \end{aligned}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$\begin{aligned} 2y'' + 5y' + 2y &= 0 \qquad\qquad y = e^{\lambda x} \\ 2\lambda^2 e^{\lambda x} + 5\lambda e^{\lambda x} + 2e^{\lambda x} &= 0 \\ 2\lambda^2 + 5\lambda + 2 &= 0 \end{aligned}$$

Find the general solution of $2y''+5y'+2y=0$.

$$\begin{aligned} 2y''+5y'+2y &= 0 \qquad\qquad y=e^{\lambda x} \\ 2\lambda^2 e^{\lambda x}+5\lambda e^{\lambda x}+2e^{\lambda x} &= 0 \\ 2\lambda^2+5\lambda+2 &= 0 \\ \lambda_{1,2} &= \frac{-5\pm\sqrt{25-16}}{4} \end{aligned}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$\begin{aligned} 2y'' + 5y' + 2y &= 0 \qquad\qquad y = e^{\lambda x} \\ 2\lambda^2 e^{\lambda x} + 5\lambda e^{\lambda x} + 2e^{\lambda x} &= 0 \\ 2\lambda^2 + 5\lambda + 2 &= 0 \\ \lambda_{1,2} &= \frac{-5 \pm \sqrt{25-16}}{4} \\ &= \frac{-5 \pm 3}{4} \end{aligned}$$

Find the general solution of $2y''+5y'+2y=0$.

$$\begin{aligned} 2y''+5y'+2y &= 0 \qquad\qquad y=e^{\lambda x} \\ 2\lambda^2e^{\lambda x}+5\lambda e^{\lambda x}+2e^{\lambda x} &= 0 \\ 2\lambda^2+5\lambda+2 &= 0 \\ \lambda_{1,2} &= \frac{-5\pm\sqrt{25-16}}{4} \\ &= \frac{-5\pm 3}{4} \\ &= -\frac{1}{2},-2 \end{aligned}$$

Find the general solution of $2y'' + 5y' + 2y = 0$.

$$
\begin{aligned}
2y'' + 5y' + 2y &= 0 \qquad\qquad y = e^{\lambda x} \\
2\lambda^2 e^{\lambda x} + 5\lambda e^{\lambda x} + 2e^{\lambda x} &= 0 \\
2\lambda^2 + 5\lambda + 2 &= 0 \\
\lambda_{1,2} &= \frac{-5 \pm \sqrt{25-16}}{4} \\
&= \frac{-5 \pm 3}{4} \\
&= -\frac{1}{2}, -2 \\
y &= c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}
\end{aligned}
$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$2y'' + 5y' + 2y \stackrel{?}{=} 0$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$2y'' + 5y' + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) \stackrel{?}{=} 0$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$2y'' + 5\left(-\frac{1}{2}c_1 e^{-\frac{1}{2}x} - 2c_2 e^{-2x}\right) + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) \stackrel{?}{=} 0$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$2\left(\frac{1}{4}c_1 e^{-\frac{1}{2}x} + 4c_2 e^{-2x}\right) + 5\left(-\frac{1}{2}c_1 e^{-\frac{1}{2}x} - 2c_2 e^{-2x}\right) + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) \stackrel{?}{=} 0$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$
\begin{aligned}
2\left(\frac{1}{4}c_1 e^{-\frac{1}{2}x} + 4c_2 e^{-2x}\right) + 5\left(-\frac{1}{2}c_1 e^{-\frac{1}{2}x} - 2c_2 e^{-2x}\right) + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) &\stackrel{?}{=} 0 \\
\frac{1}{2}c_1 e^{-\frac{1}{2}x} + 8c_2 e^{-2x} - \frac{5}{2}c_1 e^{-\frac{1}{2}x} - 10c_2 e^{-2x} + 2c_1 e^{-\frac{1}{2}x} + 2c_2 e^{-2x} &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$\begin{aligned}
2\left(\frac{1}{4}c_1 e^{-\frac{1}{2}x} + 4c_2 e^{-2x}\right) + 5\left(-\frac{1}{2}c_1 e^{-\frac{1}{2}x} - 2c_2 e^{-2x}\right) + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) &\stackrel{?}{=} 0 \\
\frac{1}{2}c_1 e^{-\frac{1}{2}x} + 8c_2 e^{-2x} - \frac{5}{2}c_1 e^{-\frac{1}{2}x} - 10c_2 e^{-2x} + 2c_1 e^{-\frac{1}{2}x} + 2c_2 e^{-2x} &\stackrel{?}{=} 0 \\
\left(\frac{1}{2} - \frac{5}{2} + 2\right) c_1 e^{-\frac{1}{2}x} &
\end{aligned}$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$
\begin{aligned}
2\left(\frac{1}{4}c_1 e^{-\frac{1}{2}x} + 4c_2 e^{-2x}\right) + 5\left(-\frac{1}{2}c_1 e^{-\frac{1}{2}x} - 2c_2 e^{-2x}\right) + 2\left(c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}\right) &\stackrel{?}{=} 0 \\
\frac{1}{2}c_1 e^{-\frac{1}{2}x} + 8c_2 e^{-2x} - \frac{5}{2}c_1 e^{-\frac{1}{2}x} - 10c_2 e^{-2x} + 2c_1 e^{-\frac{1}{2}x} + 2c_2 e^{-2x} &\stackrel{?}{=} 0 \\
\left(\frac{1}{2} - \frac{5}{2} + 2\right) c_1 e^{-\frac{1}{2}x} + (8 - 10 + 2)\, c_2 e^{-2x} &
\end{aligned}
$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$
\begin{aligned}
2\left(\frac{1}{4}c_1e^{-\frac{1}{2}x}+4c_2e^{-2x}\right)+5\left(-\frac{1}{2}c_1e^{-\frac{1}{2}x}-2c_2e^{-2x}\right)+2\left(c_1e^{-\frac{1}{2}x}+c_2e^{-2x}\right) &\stackrel{?}{=} 0\\
\frac{1}{2}c_1e^{-\frac{1}{2}x}+8c_2e^{-2x}-\frac{5}{2}c_1e^{-\frac{1}{2}x}-10c_2e^{-2x}+2c_1e^{-\frac{1}{2}x}+2c_2e^{-2x} &\stackrel{?}{=} 0\\
\left(\frac{1}{2}-\frac{5}{2}+2\right)c_1e^{-\frac{1}{2}x}+(8-10+2)\,c_2e^{-2x} &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$
\begin{aligned}
2\left(\frac{1}{4}c_1e^{-\frac{1}{2}x}+4c_2e^{-2x}\right)+5\left(-\frac{1}{2}c_1e^{-\frac{1}{2}x}-2c_2e^{-2x}\right)+2\left(c_1e^{-\frac{1}{2}x}+c_2e^{-2x}\right) &\overset{?}{=} 0\\
\frac{1}{2}c_1e^{-\frac{1}{2}x}+8c_2e^{-2x}-\frac{5}{2}c_1e^{-\frac{1}{2}x}-10c_2e^{-2x}+2c_1e^{-\frac{1}{2}x}+2c_2e^{-2x} &\overset{?}{=} 0\\
\left(\frac{1}{2}-\frac{5}{2}+2\right)c_1e^{-\frac{1}{2}x}+(8-10+2)\,c_2e^{-2x} &\overset{?}{=} 0\\
0 &= 0
\end{aligned}
$$

Does $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-2x}$ Really Solve $2y'' + 5y' + 2y = 0$?

$$
\begin{aligned}
2\left(\frac{1}{4}c_1e^{-\frac{1}{2}x}+4c_2e^{-2x}\right)+5\left(-\frac{1}{2}c_1e^{-\frac{1}{2}x}-2c_2e^{-2x}\right)+2\left(c_1e^{-\frac{1}{2}x}+c_2e^{-2x}\right) &\stackrel{?}{=} 0\\
\frac{1}{2}c_1e^{-\frac{1}{2}x}+8c_2e^{-2x}-\frac{5}{2}c_1e^{-\frac{1}{2}x}-10c_2e^{-2x}+2c_1e^{-\frac{1}{2}x}+2c_2e^{-2x} &\stackrel{?}{=} 0\\
\left(\frac{1}{2}-\frac{5}{2}+2\right)c_1e^{-\frac{1}{2}x}+(8-10+2)\,c_2e^{-2x} &\stackrel{?}{=} 0\\
0 &= 0
\end{aligned}
$$

$\surd$

Find the general solution of $y'' + 4y' + 5y = 0$.

Find the general solution of $y'' + 4y' + 5y = 0$.

$$y'' + 4y' + 5y = 0$$

Find the general solution of $y'' + 4y' + 5y = 0$.

$$\begin{aligned} y'' + 4y' + 5y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 5e^{\lambda x} &= 0 \end{aligned}$$

Find the general solution of $y'' + 4y' + 5y = 0$.

$$\begin{aligned} y'' + 4y' + 5y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 5e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda + 5 &= 0 \end{aligned}$$

Find the general solution of $y'' + 4y' + 5y = 0$.

$$\begin{aligned} y'' + 4y' + 5y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 5e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda + 5 &= 0 \\ \lambda_{1,2} &= \frac{-4 \pm \sqrt{16 - 20}}{2} \end{aligned}$$

Find the general solution of $y'' + 4y' + 5y = 0$.

$$\begin{aligned} y'' + 4y' + 5y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 5e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda + 5 &= 0 \\ \lambda_{1,2} &= \frac{-4 \pm \sqrt{16-20}}{2} \\ &= -2 \pm i \end{aligned}$$

Find the general solution of $y'' + 4y' + 5y = 0$.

$$
\begin{aligned}
y'' + 4y' + 5y &= 0 \\
\lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 5e^{\lambda x} &= 0 \\
\lambda^2 + 4\lambda + 5 &= 0 \\
\lambda_{1,2} &= \frac{-4 \pm \sqrt{16-20}}{2} \\
&= -2 \pm i
\end{aligned}
$$

$$
y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)
$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$y'' + 4y' + 5y \stackrel{?}{=} 0$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$
\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = c_1e^{-2x}\cos(x) + c_2e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$
\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$
\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
(3 - 8 + 5)\, e^{-2x}\cos(x) &
\end{aligned}
$$

Does $y = c_1e^{-2x}\cos(x) + c_2e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
(3-8+5)\,e^{-2x}\cos(x) + (4-4)\,e^{-2x}\sin(x) &\stackrel{?}{=} 0
\end{aligned}$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
(3-8+5)\,e^{-2x}\cos(x) + (4-4)\,e^{-2x}\sin(x) &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$
\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
(3-8+5)\,e^{-2x}\cos(x) + (4-4)\,e^{-2x}\sin(x) &\stackrel{?}{=} 0 \\
0 &= 0 \quad \surd
\end{aligned}
$$

Does $y = c_1 e^{-2x}\cos(x) + c_2 e^{-2x}\sin(x)$ Really Solve $y'' + 4y' + 5y = 0$?

$$\begin{aligned}
y'' + 4y' + 5y &\stackrel{?}{=} 0 \\
\left(e^{-2x}\cos(x)\right)'' + 4\left(e^{-2x}\cos(x)\right)' + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
\left(3e^{-2x}\cos(x) + 4e^{-2x}\sin(x)\right) & \\
+4\left(-2e^{-2x}\cos(x) - e^{-2x}\sin(x)\right) + 5\left(e^{-2x}\cos(x)\right) &\stackrel{?}{=} 0 \\
(3 - 8 + 5)\, e^{-2x}\cos(x) + (4 - 4)\, e^{-2x}\sin(x) &\stackrel{?}{=} 0 \\
0 &= 0 \quad \surd
\end{aligned}$$

Check $y = e^{-2x}\sin(x)$ yourself.

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$y'' + 4y' + 4y = 0$$

Solve the Initial Value Problem $y''+4y'+4y=0$, $y(0)=1$, $y'(0)=0$.

$$\begin{aligned} y''+4y'+4y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0 \end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$
\begin{aligned}
y'' + 4y' + 4y &= 0 \\
\lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \\
\lambda^2 + 4\lambda + 4 &= 0
\end{aligned}
$$

Solve the Initial Value Problem $y''+4y'+4y=0$, $y(0)=1$, $y'(0)=0$.

$$\begin{aligned} y''+4y'+4y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0 \\ \lambda^2+4\lambda+4 &= 0 \\ \lambda_{1,2} &= \frac{-4\pm\sqrt{16-16}}{2} \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=0$, $y(0)=1$, $y'(0)=0$.

$$\begin{aligned}
y''+4y'+4y &= 0\\
\lambda^2e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0\\
\lambda^2+4\lambda+4 &= 0\\
\lambda_{1,2} &= \frac{-4\pm\sqrt{16-16}}{2}\\
&= -2
\end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$
\begin{aligned}
y'' + 4y' + 4y &= 0 \\
\lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \\
\lambda^2 + 4\lambda + 4 &= 0 \\
\lambda_{1,2} &= \frac{-4 \pm \sqrt{16-16}}{2} \\
&= -2
\end{aligned}
$$

$$
y = c_1 e^{-2x} + c_2 x e^{-2x}
$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$y = c_1 e^{-2x} + c_2 x e^{-2x}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$\begin{aligned} y &= c_1 e^{-2x} + c_2 x e^{-2x} \\ y' &= -2c_1 e^{-2x} + c_2 \left(e^{-2x} - 2x e^{-2x} \right) \end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$\begin{aligned} y &= c_1 e^{-2x} + c_2 x e^{-2x} \\ y' &= -2c_1 e^{-2x} + c_2 \left(e^{-2x} - 2xe^{-2x} \right) \end{aligned}$$

$$1 = c_1$$

Solve the Initial Value Problem $y''+4y'+4y=0$, $y(0)=1$, $y'(0)=0$.

$$\begin{aligned} y &= c_1e^{-2x}+c_2xe^{-2x} \\ y' &= -2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right) \end{aligned}$$

$$\begin{aligned} 1 &= c_1 \\ 0 &= -2c_1+c_2 \end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$\begin{aligned} y &= c_1 e^{-2x} + c_2 x e^{-2x} \\ y' &= -2c_1 e^{-2x} + c_2 \left(e^{-2x} - 2x e^{-2x} \right) \end{aligned}$$

$$\begin{aligned} 1 &= c_1 \\ 0 &= -2c_1 + c_2 \\ c_2 &= 2c_1 = 2 \end{aligned}$$

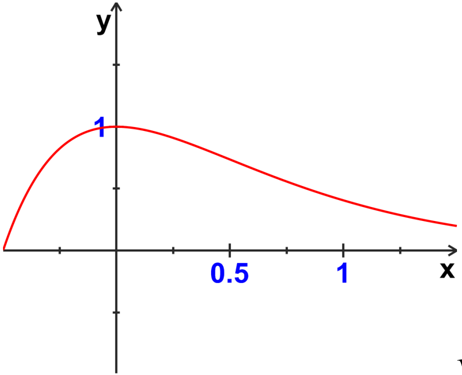
Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

$$
\begin{aligned}
y &= c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= -2c_1 e^{-2x} + c_2 \left(e^{-2x} - 2x e^{-2x} \right)
\end{aligned}
$$

$$
\begin{aligned}
1 &= c_1 \\
0 &= -2c_1 + c_2 \\
c_2 &= 2c_1 = 2
\end{aligned}
$$

$$
y = e^{-2x} + 2x e^{-2x}
$$

Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.



$$y = e^{-2x} + 2xe^{-2x}$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$y'' + 4y' + 4y \stackrel{?}{=} 0$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y'' + 4y' + 4y &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-2e^{-2x} + 2e^{-2x} - 4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y'' + 4y' + 4y &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-2e^{-2x} + 2e^{-2x} - 4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y'' + 4y' + 4y &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-2e^{-2x} + 2e^{-2x} - 4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
(-4+4)\,e^{-2x} + (8-16+8)\,xe^{-2x} &\stackrel{?}{=} 0
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y'' + 4y' + 4y &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-2e^{-2x} + 2e^{-2x} - 4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
(-4+4)\,e^{-2x} + (8-16+8)\,xe^{-2x} &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y'' + 4y' + 4y &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-2e^{-2x} + 2e^{-2x} - 4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
\left(-4e^{-2x} + 8xe^{-2x}\right) & \\
+4\left(-4xe^{-2x}\right) + 4\left(e^{-2x} + 2xe^{-2x}\right) &\stackrel{?}{=} 0 \\
(-4+4)\,e^{-2x} + (8-16+8)\,xe^{-2x} &\stackrel{?}{=} 0 \\
0 &= 0 \quad \checkmark
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$y = e^{-2x} + 2xe^{-2x}$$

Does $y=e^{-2x}+2xe^{-2x}$ Solve the Initial Value Problem $y''+4y'+4y=0$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned} y &= e^{-2x}+2xe^{-2x} \\ y(0) &= 1 \end{aligned}$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} y &= e^{-2x} + 2xe^{-2x} \\ y(0) &= 1 \quad \surd \end{aligned}$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y &= e^{-2x} + 2xe^{-2x} \\
y(0) &= 1 \quad \surd \\
y' &= -2e^{-2x} + 2\left(e^{-2x} - 2xe^{-2x}\right)
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y &= e^{-2x} + 2xe^{-2x} \\
y(0) &= 1 \quad \surd \\
y' &= -2e^{-2x} + 2\left(e^{-2x} - 2xe^{-2x}\right) \\
y'(0) &= -2 + 2 = 0
\end{aligned}
$$

Does $y = e^{-2x} + 2xe^{-2x}$ Solve the Initial Value Problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y &= e^{-2x} + 2xe^{-2x} \\
y(0) &= 1 \quad \surd \\
y' &= -2e^{-2x} + 2\left(e^{-2x} - 2xe^{-2x}\right) \\
y'(0) &= -2 + 2 = 0 \quad \surd
\end{aligned}
$$