Matrix Multiplication

Bernd Schröder

Matrix Multiplication (Informally)

Matrix Multiplication (Informally)

1. Matrix addition is componentwise and so is multiplication with a scalar.

Matrix Multiplication (Informally)

1. Matrix addition is componentwise and so is multiplication with a scalar.
2. Matrix multiplication is more complicated.

Matrix Multiplication (Informally)

1. Matrix addition is componentwise and so is multiplication with a scalar.
2. Matrix multiplication is more complicated.
3. Take 1: To multiply an $m \times n$ matrix with a vector with n rows, for each row of the matrix, go along the row, multiply each entry with the corresponding entry of the vector. Add these products and enter the result in the corresponding row of the resulting vector.

Matrix Times Vector

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \left(\vphantom{\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}} \right.$$

Matrix Times Vector

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \Bigg($$

Matrix Times Vector

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \left(\vphantom{\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}} \right.$$

Matrix Times Vector

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & \boxed{2} & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & \boxed{2} & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & \boxed{2} & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 2\cdot(-1) \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 2\cdot(-1) \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & \boxed{3} \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3+2\cdot(-1) \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & \boxed{3} \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 2\cdot(-1) \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & \boxed{3} \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 1\cdot 3+2\cdot(-1)+3\cdot 4 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 2\cdot(-1) + 3\cdot 4 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ \boxed{1} & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ \boxed{1} & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ \boxed{1} & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & \boxed{4} & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & \boxed{4} & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 \\ \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & \boxed{4} & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 + 4 \cdot (-1) \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 + 4 \cdot (-1) \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & \boxed{-1} \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 + 4 \cdot (-1) \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & \boxed{-1} \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ 1\cdot 3 + 4\cdot(-1) \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & \boxed{-1} \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \cdot 3 + 4 \cdot (-1) + (-1) \cdot 4 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & \boxed{-1} \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ \end{pmatrix}$$

Matrix Times Vector

$$\left(\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{array}\right)\left(\begin{array}{c} 3 \\ -1 \\ 4 \end{array}\right)=\left(\begin{array}{c} 13 \\ -5 \\ \\ \end{array}\right.$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ \boxed{3} & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ \boxed{3} & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ \boxed{3} & 2 & 4 \end{pmatrix} \begin{pmatrix} \boxed{3} \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3 \cdot 3 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & \boxed{2} & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3 \cdot 3 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & \boxed{2} & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3 \cdot 3 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & \boxed{2} & 4 \end{pmatrix} \begin{pmatrix} 3 \\ \boxed{-1} \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3\cdot 3+2\cdot(-1) \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3\cdot 3+2\cdot(-1) \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & \boxed{4} \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3\cdot 3+2\cdot(-1) \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & \boxed{4} \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3\cdot 3+2\cdot(-1) \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & \boxed{4} \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 3\cdot 3 + 2\cdot(-1) + 4\cdot 4 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & \boxed{4} \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ \boxed{4} \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 23 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 23 \end{pmatrix}$$

Matrix Times Vector

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ 23 \end{pmatrix}$$

Matrix Multiplication (Formally)

Matrix Multiplication (Formally)

1. Let m,n,p be positive integers. For the real $m\times n$ matrix $A=\left(a_{jk}\right)_{\substack{j=1,\ldots,m\\ k=1,\ldots,n}}$ and the real $p\times m$ matrix $B=\left(b_{ij}\right)_{\substack{i=1,\ldots,p\\ j=1,\ldots,m}}$, we define the **product** to be the real $p\times n$ matrix $BA:=\left(\sum_{j=1}^{m} b_{ij}a_{jk}\right)_{\substack{i=1,\ldots,p\\ k=1,\ldots,n}}$.

Matrix Multiplication (Formally)

1. Let m,n,p be positive integers. For the real $m\times n$ matrix $A=\left(a_{jk}\right)_{\substack{j=1,\ldots,m\\ k=1,\ldots,n}}$ and the real $p\times m$ matrix $B=\left(b_{ij}\right)_{\substack{i=1,\ldots,p\\ j=1,\ldots,m}}$, we define the **product** to be the real $p\times n$ matrix $BA:=\left(\sum_{j=1}^{m} b_{ij}a_{jk}\right)_{\substack{i=1,\ldots,p\\ k=1,\ldots,n}}$.
2. Informally, the right matrix A is a collection of column vectors. We multiply the left matrix B with each of the columns of A and report the results as columns next to each other.

Matrix Times Matrix

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} \boxed{3 \quad -1 \quad 2} \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} \boxed{3 \quad -1 \quad 2} \\ -4 \quad 0 \quad 1 \\ -2 \quad 2 \quad 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} \boxed{3 \quad -1 \quad 2} \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ \boxed{-4 \quad 0 \quad 1} \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ \boxed{-2 \quad 2 \quad 3} \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ \boxed{-2 \quad 2 \quad 3} \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & 6 & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & 6 & \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & 6 & -6 \end{pmatrix}$$

Matrix Times Matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -4 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 7 & 1 & 0 \\ 1 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & -17 \\ -7 & -2 & 8 \\ 13 & 6 & -6 \end{pmatrix}$$