The Method of Frobenius

Bernd Schröder

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2. But P and Q cannot be arbitrary: $(x - x_0)P(x)$ and $(x - x_0)^2 Q(x)$ must be analytic at x_0.

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1. The method of Frobenius works for differential equations of the form $y''+P(x)y'+Q(x)y=0$ in which P or Q is not analytic at the point of expansion x_0.
2. But P and Q cannot be arbitrary: $(x-x_0)P(x)$ and $(x-x_0)^2Q(x)$ must be analytic at x_0.
3. Instead of a series solution $y=\sum_{n=0}^{\infty}c_n(x-x_0)^n$, we obtain a solution of the form $y=\sum_{n=0}^{\infty}c_n(x-x_0)^{n+r}$.

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1. The method of Frobenius works for differential equations of the form $y'' + P(x)y' + Q(x)y = 0$ in which P or Q is not analytic at the point of expansion x_0.
2. But P and Q cannot be arbitrary: $(x - x_0)P(x)$ and $(x - x_0)^2 Q(x)$ must be analytic at x_0.
3. Instead of a series solution $y = \sum_{n=0}^{\infty} c_n (x - x_0)^n$, we obtain a solution of the form $y = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}$.
4. The method of Frobenius is guaranteed to produce one solution, but it may not produce two linearly independent solutions.

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That's it.

Solve the Differential Equation

$$9x^2y'' + 3x^2y' + 2y = 0$$

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Solve the Differential Equation $9x^2y''+3x^2y'+2y=0$

$$\begin{aligned} 9x^2y''+3x^2y'+2y &= 0 \\ 9x^2\left(\sum_{n=0}^{\infty} c_n x^{n+r}\right)''+3x^2\left(\sum_{n=0}^{\infty} c_n x^{n+r}\right)'+2\left(\sum_{n=0}^{\infty} c_n x^{n+r}\right) &= 0 \end{aligned}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

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\begin{aligned}
9x^2y'' + 3x^2y' + 2y &= 0 \\
9x^2\left(\sum_{n=0}^{\infty} c_n x^{n+r}\right)'' + 3x^2\sum_{n=0}^{\infty} c_n (n+r)x^{n+r-1} + 2\sum_{n=0}^{\infty} c_n x^{n+r} &= 0
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\sum_{n=0}^{\infty} 9(n+r)(n+r-1)c_nx^{n+r} + \sum_{n=0}^{\infty} 3(n+r)c_nx^{n+r+1} + \sum_{n=0}^{\infty} 2c_nx^{n+r} &= 0
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\sum_{n=0}^{\infty} 9(n+r)(n+r-1)c_n x^{n+r} + \sum_{n=0}^{\infty} 3(n+r)c_n x^{n+r+1} + \sum_{n=0}^{\infty} 2c_n x^{n+r} &= 0 \\
\sum_{k=0}^{\infty} 9(k+r)(k+r-1)c_k x^{k+r} + \sum_{k=1}^{\infty} 3(k+r-1)c_{k-1} x^{k+r} + \sum_{k=0}^{\infty} 2c_k x^{k+r} &= 0
\end{aligned}$$

Solve the Differential Equation $9x^2y''+3x^2y'+2y=0$

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\begin{aligned}
9x^2y''+3x^2y'+2y &= 0\\
9x^2\sum_{n=0}^{\infty}c_n(n+r)(n+r-1)x^{n+r-2}+3x^2\sum_{n=0}^{\infty}c_n(n+r)x^{n+r-1}+2\sum_{n=0}^{\infty}c_nx^{n+r} &= 0\\
\sum_{n=0}^{\infty}9(n+r)(n+r-1)c_nx^{n+r}+\sum_{n=0}^{\infty}3(n+r)c_nx^{n+r+1}+\sum_{n=0}^{\infty}2c_nx^{n+r} &= 0\\
\sum_{k=0}^{\infty}9(k+r)(k+r-1)c_kx^{k+r}+\sum_{k=1}^{\infty}3(k+r-1)c_{k-1}x^{k+r}+\sum_{k=0}^{\infty}2c_kx^{k+r} &= 0\\
\big(9r(r-1)+2\big)c_0x^r+\sum_{k=1}^{\infty}\Big[9(k+r)(k+r-1)c_k+3(k+r-1)c_{k-1}+2c_k\Big]x^{k+r} &= 0
\end{aligned}
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Indicial equation:

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$$\begin{aligned} \big(9r(r-1)+2\big)c_0 &= 0 \\ 9r(r-1)+2 &= 0 \\ 9r^2-9r+2 &= 0 \\ r_{1,2} &= \frac{9 \pm \sqrt{81-72}}{18} \end{aligned}$$

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Recurrence Relation.

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Recurrence Relation.

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Recurrence Relation.

$$\begin{aligned} 9(k+r)(k+r-1)c_k + 3(k+r-1)c_{k-1} + 2c_k &= 0 \\ \left[9(k+r)(k+r-1)+2\right]c_k + 3(k+r-1)c_{k-1} &= 0 \end{aligned}$$

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Recurrence Relation.

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\begin{aligned}
9(k+r)(k+r-1)c_k + 3(k+r-1)c_{k-1} + 2c_k &= 0 \\
\left[9(k+r)(k+r-1)+2\right]c_k + 3(k+r-1)c_{k-1} &= 0 \\
\left[9(k+r)(k+r-1)+2\right]c_k &= -3(k+r-1)c_{k-1}
\end{aligned}
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\Big[9(k+r)(k+r-1)+2\Big]c_k &= -3(k+r-1)c_{k-1} \\
c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1}
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Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

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$$c_k = \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1}$$

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&= \frac{1-3k}{(3k+2)(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1}
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Solve the Differential Equation $9x^2y''+3x^2y'+2y=0$

$$r=\frac{2}{3}\,,\ c_0=1$$

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c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1}=\frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1}\\
&= \frac{1-3k}{(3k+2)\,(3k-1)+2}c_{k-1}=\frac{1-3k}{9k^2+3k}c_{k-1}\\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1}
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&= \frac{1-3k}{(3k+2)\,(3k-1)+2}c_{k-1}=\frac{1-3k}{9k^2+3k}c_{k-1}\\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1}=-\frac{2}{12}\cdot 1
\end{aligned}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

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\begin{aligned}
c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2} c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2} c_{k-1} \\
&= \frac{1-3k}{(3k+2)(3k-1)+2} c_{k-1} = \frac{1-3k}{9k^2+3k} c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1} c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6}
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c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1} \\
&= \frac{1-3k}{(3k+2)(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6} \\
c_2 &= \frac{1-3\cdot 2}{9\cdot 2^2+3\cdot 2}c_{2-1}
\end{aligned}$$

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&= \frac{1-3k}{(3k+2)(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6} \\
c_2 &= \frac{1-3\cdot 2}{9\cdot 2^2+3\cdot 2}c_{2-1} = -\frac{5}{42}\left(-\frac{1}{6}\right)
\end{aligned}
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$$r = \frac{2}{3}\,,\ c_0 = 1$$

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&= \frac{1-3k}{(3k+2)(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6} \\
c_2 &= \frac{1-3\cdot 2}{9\cdot 2^2+3\cdot 2}c_{2-1} = -\frac{5}{42}\left(-\frac{1}{6}\right) = \frac{5}{252}
\end{aligned}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$r = \frac{2}{3}$, $c_0 = 1$

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c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1} \\
&= \frac{1-3k}{(3k+2)\,(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6} \\
c_2 &= \frac{1-3\cdot 2}{9\cdot 2^2+3\cdot 2}c_{2-1} = -\frac{5}{42}\left(-\frac{1}{6}\right) = \frac{5}{252} \\
c_3 &= \frac{1-3\cdot 3}{9\cdot 3^2+3\cdot 3}c_{3-1}
\end{aligned}$$

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c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1} \\
&= \frac{1-3k}{(3k+2)(3k-1)+2}c_{k-1} = \frac{1-3k}{9k^2+3k}c_{k-1} \\
c_1 &= \frac{1-3\cdot 1}{9\cdot 1^2+3\cdot 1}c_{1-1} = -\frac{2}{12}\cdot 1 = -\frac{1}{6} \\
c_2 &= \frac{1-3\cdot 2}{9\cdot 2^2+3\cdot 2}c_{2-1} = -\frac{5}{42}\left(-\frac{1}{6}\right) = \frac{5}{252} \\
c_3 &= \frac{1-3\cdot 3}{9\cdot 3^2+3\cdot 3}c_{3-1} = -\frac{8}{90}\frac{5}{252}
\end{aligned}
$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}, \; c_0 = 1$$

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c_k &= \frac{3-3k-3r}{9(k+r)(k+r-1)+2}c_{k-1} = \frac{3-3k-3\cdot\frac{2}{3}}{3\left(k+\frac{2}{3}\right)3\left(k+\frac{2}{3}-1\right)+2}c_{k-1} \\
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c_3 &= \frac{1-3\cdot 3}{9\cdot 3^2+3\cdot 3}c_{3-1} = -\frac{8}{90}\frac{5}{252} = -\frac{1}{567}
\end{aligned}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}$$

$$c_4 = \frac{1 - 3\cdot 4}{9\cdot 4^2 + 3\cdot 4} c_{4-1}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}$$

$$c_4 = \frac{1-3\cdot 4}{9\cdot 4^2+3\cdot 4}c_{4-1} = -\frac{11}{156}\left(-\frac{1}{567}\right)$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}$$

$$c_4 = \frac{1-3\cdot 4}{9\cdot 4^2+3\cdot 4}c_{4-1} = -\frac{11}{156}\left(-\frac{1}{567}\right) = \frac{11}{88452}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}$$

$$\begin{aligned} c_4 &= \frac{1-3\cdot 4}{9\cdot 4^2+3\cdot 4}c_{4-1} = -\frac{11}{156}\left(-\frac{1}{567}\right) = \frac{11}{88452} \\ y_1 &= x^{\frac{2}{3}} - \frac{1}{6}x^{\frac{2}{3}+1} + \frac{5}{252}x^{\frac{2}{3}+2} - \frac{1}{567}x^{\frac{2}{3}+3} + \frac{11}{88452}x^{\frac{2}{3}+4} \end{aligned}$$

Solve the Differential Equation $9x^2y'' + 3x^2y' + 2y = 0$

$$r = \frac{2}{3}$$

$$\begin{aligned}
c_4 &= \frac{1-3\cdot 4}{9\cdot 4^2+3\cdot 4}c_{4-1} = -\frac{11}{156}\left(-\frac{1}{567}\right) = \frac{11}{88452} \\
y_1 &= x^{\frac{2}{3}} - \frac{1}{6}x^{\frac{2}{3}+1} + \frac{5}{252}x^{\frac{2}{3}+2} - \frac{1}{567}x^{\frac{2}{3}+3} + \frac{11}{88452}x^{\frac{2}{3}+4} \\
&= x^{\frac{2}{3}} - \frac{1}{6}x^{\frac{5}{3}} + \frac{5}{252}x^{\frac{8}{3}} - \frac{1}{567}x^{\frac{11}{3}} + \frac{11}{88452}x^{\frac{14}{3}}
\end{aligned}$$

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$$r = \frac{1}{3}$$

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$$y1(x) := x^{\frac{2}{3}} - \frac{1}{6}\cdot x^{\frac{5}{3}} + \frac{5}{252}\cdot x^{\frac{8}{3}} - \frac{1}{567}\cdot x^{\frac{11}{3}} + \frac{11}{88452}x^{\frac{14}{3}}$$

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$$9\cdot x^2\cdot\frac{d^2}{dx^2}y2(x) + 3\cdot x^2\cdot\frac{d}{dx}y2(x) + 2\cdot y2(x) \text{ collect},x \rightarrow \frac{91}{42768}\cdot x^{\frac{16}{3}}$$

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And, of course, we use a computer algebra system.