Modeling and Solving the Mixing of Liquids

Bernd Schröder

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3. The main idea for the setup is to track the *total amount* of the dissolved substance.
4. When we do this, the rate of change of the total amount is simply the inflow minus the outflow.
5. Unit checks help prevent mistakes.

A $1000l$ vat initially contains brine in which $7kg$ of salt are dissolved. Brine with a salt content of $\frac{3}{1000}\frac{kg}{l}$ enters the vat at a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

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7kg = S(0) &= 3kg - ke^{-\frac{1}{100min}0} = 3kg - k \\
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Answering the Questions

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$$\begin{aligned} 7kg = S(0) &= 3kg - ke^{-\frac{1}{100min}0}\,kg = 3kg - k\,kg \\ k &= -4 \\ S(t) &= 3kg + 4kge^{-\frac{1}{100min}t} \end{aligned}$$