

Modeling and Solving the Mixing of Liquids

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Underlying Assumptions and Main Ideas

1. We will assume that the mixture is "thoroughly stirred".

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- 2. Of course this is not entirely realistic, but it leads to solvable equations, which describe the process quite well.
- 3. The main idea for the setup is to track the *total amount* of the dissolved substance.

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- 2. Of course this is not entirely realistic, but it leads to solvable equations, which describe the process quite well.
- 3. The main idea for the setup is to track the *total amount* of the dissolved substance.
- 4. When we do this, the rate of change of the total amount is simply the inflow minus the outflow.

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- 2. Of course this is not entirely realistic, but it leads to solvable equations, which describe the process quite well.
- 3. The main idea for the setup is to track the *total amount* of the dissolved substance.
- 4. When we do this, the rate of change of the total amount is simply the inflow minus the outflow.
- 5. Unit checks help prevent mistakes.

	in Example	Double Cheen	enno neprioed
A di	1000 <i>l</i> vat initially contains brine is solved. Brine with a salt content	In which $7kg$ of salt are of $\frac{3}{1000} \frac{kg}{l}$ enters the value	at at
a th	rate of $10 \frac{l}{min}$. The thoroughly mine same rate.	xed solution exits the va	at at

Double Check

Units Reprised

An Example

Overview	An Example	Double Check	Units Reprised
A 1000 <i>l</i> vat ini dissolved. Brir	tially contains brine in the with a salt content of	which $7kg$ of salt are of $\frac{3}{1000}\frac{kg}{l}$ enters the values	at at
a rate of $10 \frac{t}{mir}$ the same rate. Determine the	$\frac{1}{2}$. The thoroughly mix	ed solution exits the va vat after 10 <i>min</i> .	it at

verv	view An Example	Double Check	Units Reprised
	A 1000 <i>l</i> vat initially contain	ns brine in which $7kg$ of s	alt are
	dissolved. Brine with a salt	content of $\frac{5}{1000} \frac{kg}{l}$ enter	s the vat at
	a rate of $10\frac{l}{min}$. The thorous	ughly mixed solution exits	s the vat at
	the same rate.		
	Determine the amount of sa	alt in the vat after 10min.	
	D		

Determine the long term concentration of salt in the brine.

vervi	ew An Example	Double Check	Units Reprised
	A 1000 <i>l</i> vat initially contains brine in	which $7kg$ of salt are $3 kg$	
	dissolved. Brine with a salt content of <i>l</i>	$f = \frac{1}{1000} \frac{l}{l}$ enters the value of $\frac{1}{1000} \frac{l}{l}$	at at
	a rate of $10\frac{1}{min}$. The thoroughly mixed	ed solution exits the va	nt at
	the same rate.		
	Determine the amount of salt in the v	at after 10min.	

Determine the long term concentration of salt in the brine.

S(0)

Overview	An Example	Double Check	Units Reprised	
А	1000l vat initially contains brine in	n which 7kg of salt are		
d	ssolved. Brine with a salt content of	of $\frac{3}{3} \frac{kg}{kg}$ enters the v	at at	

a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

 $\frac{dS}{dt}$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{\rm in}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{\rm in} - F_{\rm out}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{\rm in} - F_{\rm out}$$
$$= \frac{3}{1000} \frac{kg}{l}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{\rm in} - F_{\rm out}$$
$$= \frac{3}{1000} \frac{kg}{l} \cdot 10 \frac{l}{min}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{in} - F_{out}$$

$$= \frac{3}{1000} \frac{kg}{l} \cdot 10 \frac{l}{min} - 10 \frac{l}{min}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

$$S(0) = 7kg$$

$$\frac{dS}{dt} = F_{in} - F_{out}$$

$$= \frac{3}{1000} \frac{kg}{l} \cdot 10 \frac{l}{min} - 10 \frac{l}{min} \cdot \frac{S(t)}{1000l}$$

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a rate of $10\frac{l}{min}$. The thoroughly mixed solution exits the vat at the same rate.

Determine the amount of salt in the vat after 10min.

Determine the long term concentration of salt in the brine.

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Overview	An Example			Do	ouble Check		Units Reprised
Solve the Initial	Value Problem	$\frac{dS}{dt} =$	$\frac{3}{100}$	kg min	$-\frac{S(t)}{100}$	$\frac{1}{\min}, S(0) = 7$	kg

Overview	An Example	Dou	ible Check	Units Reprised
Solve the Initial Value	e Problem $\frac{dS}{dt}$	$\frac{3}{100} = \frac{3}{100} \frac{kg}{min} - \frac{3}{100} \frac{kg}{min}$	$-\frac{S(t)}{100}\frac{1}{min}, S(0) = 7k$	g
	$\frac{dS}{dt} = \frac{1}{2}$	$\frac{3}{100}\frac{kg}{min} - \frac{S}{100}$	$\frac{(t)}{00}\frac{1}{min}$	

Overview A	An Example	I	Jouble Check	Units Reprised
Solve the Initial Value	Problem $\frac{dx}{d}$	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min}$	$\frac{S(t)}{100}\frac{1}{min}, S(0) =$	= 7kg
	$\frac{dS}{dt} = -$	$\frac{3}{100}\frac{kg}{min} - \frac{kg}{kg}$	$\frac{S(t)}{100} \frac{1}{min} = \frac{3kg}{100k}$	$\frac{S(t)}{min}$

Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{dS}{dt}$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min}$, S(0) = 7kg
3	$\frac{dS}{dt} = \frac{3}{10}$ $\frac{dS}{kg - S(t)} = \frac{1}{10}$	$\frac{\frac{3}{00}}{\frac{kg}{min}} - \frac{S(t)}{100}\frac{1}{min} = \frac{1}{\frac{1}{00min}} dt$	$\frac{3kg - S(t)}{100min}$

Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{ds}{dt}$	$\frac{S}{t} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min},$	S(0) = 7kg
$\int \frac{1}{3kg}$	$\frac{dS}{dt} = \frac{dS}{dt}$ $\frac{dS}{3kg - S(t)} = \frac{1}{-S(t)} dS = \frac{1}{2}$	$\frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min} =$ $\frac{1}{100min} dt$ $\int \frac{1}{100min} dt$	$\frac{3kg - S(t)}{100min}$

Overview	An Example	Double Check	Units Reprised
Solve the Initi	al Value Problem $\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min}$, $S(0) = 7kg$
	dS 3	kg = S(t) = 1	3kg - S(t)
	$\frac{dt}{dt} = \frac{100}{100}$	$\frac{1}{100}\frac{1}{100}\frac{1}{100}min =$	<u>100min</u>
	dS	1	
	$\frac{1}{3kg - S(t)} = \frac{1}{100}$	<u>min</u> at	
$\int \frac{1}{3k}$	$\frac{1}{ag - S(t)} dS = \int \frac{1}{1}$	$\frac{1}{100min} dt$	
$-\ln$	$\left 3kg - S(t)\right = \frac{100}{100}$	$\frac{1}{min}t+c$	

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Overview	An Example	Double Check	Units Reprised
Solve the Initia	al Value Problem	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0) = 7k_0$	g
	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min} = \frac{3kg - S(t)}{100min}$)
	dS	1	
	$\frac{3kg - S(t)}{3kg - S(t)} =$	$\frac{1}{100min}$ dt	
$\int \frac{1}{3k}$	$\frac{1}{sg-S(t)} dS =$	$\int \frac{1}{100min} dt$	
$-\ln$	3kg - S(t) =	$\frac{1}{100min}t+c$	
	3kg - S(t) =	$e^{-\frac{1}{100min}t-c}$	

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Overview	An Example	Double Check	Units Reprised
Solve the Initia	al Value Problem	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0) =$	=7kg
	$\frac{dS}{dS}$ _	3 kg S(t) 1 - 3kg -	-S(t)
	dt –	100 min 100 min 100	min
	dS	1 dt	
	$\overline{3kg - S(t)} =$	$\frac{100min}{100min}$ at	
$\int \frac{1}{3k}$	$\frac{1}{sg - S(t)} dS =$	$\int \frac{1}{100min} dt$	
$-\ln$	3kg-S(t) =	$\frac{1}{100min}t+c$	
	3kg - S(t) =	$e^{-\frac{1}{100\min}t-c} = ke^{-\frac{1}{100\min}t}$	

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Overview	An Example	Double Check	Units Reprised
Solve the Initi	al Value Problem	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0) = 7kg$	3
	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min} = \frac{3kg - S(t)}{100min}$	-
	dS	1	
	$\overline{3kg - S(t)} =$	$\frac{100min}{100min}$ dt	
$\int \frac{1}{3k}$	$\frac{1}{kg - S(t)} dS =$	$\int \frac{1}{100min} dt$	
$-\ln$	$\left 3kg - S(t)\right =$	$\frac{1}{100min}t+c$	
	3kg - S(t) =	$e^{-\frac{1}{100min}t-c} = ke^{-\frac{1}{100min}t}$	
	S(t) =	$3kg - ke^{-\frac{1}{100min}t}$	

Overview	An Example			Do	ouble Check		Units Reprised
Solve the Initial	Value Problem	$\frac{dS}{dt} =$	$\frac{3}{100}$	kg min	$-\frac{S(t)}{100}$	$\frac{1}{\min}, S(0) = 7$	kg

Overview	An Example			Dout	ole Check		Units Reprised
Solve the Initial	Value Problem	$\frac{dS}{dt} =$	$\frac{3}{100}$	<u>kg</u> min	$\frac{S(t)}{100}\frac{1}{n}$	$\frac{1}{nin}, S(0) = 7k_{\xi}$	3
-	7kg = S(0)						

Overview	An Example	Double Check	Units Reprised
Solve the Initia	al Value Problem $\frac{dS}{dt} =$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(t)$	0) = 7kg
	7kg = S(0) = 3k	$g - ke^{-\frac{1}{100min}0}$	

Overview	An Example	Double Check	Units Reprised
Solve the Initi	al Value Problem $\frac{dS}{dt} =$	$= \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0)$	=7kg
	7kg = S(0) = 3k	$kg - ke^{-\frac{1}{100min}0} = 3kg - k$	

Overview	An Example	Doub	le Check	Units Reprised
Solve the Initial Value	e Problem $\frac{dS}{dt} = \frac{1}{1}$	$\frac{3}{00}\frac{kg}{min}$ –	$\frac{S(t)}{100}\frac{1}{min}, S(0) = 7k_0$	g
7 <i>kg</i> =	= S(0) = 3kg + kg	$-ke^{-\frac{1}{100m}}$	$\overline{m}^0 = 3kg - k$	

Overview	An Example	Double Check	Units Reprised
Solve the Initia	l Value Problem $\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min}, S(0)$	=7kg
	7kg = S(0) = 3kg $k = -4$	$g - ke^{-\frac{1}{100min}0} = 3kg - k$ kg	
	S(t) = 3kg	$g + 4kge^{-\frac{1}{100min}t}$	

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S(10min)

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$

= $3kg + 4kge^{-\frac{1}{10}}$

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$

= $3kg + 4kge^{-\frac{1}{10}} \approx 6.6193kg$

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$
$$= 3kg + 4kge^{-\frac{1}{10}} \approx 6.6193kg$$
$$\lim_{t \to \infty} \frac{S(t)}{1000l}$$

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$

= $3kg + 4kge^{-\frac{1}{10}} \approx 6.6193kg$
$$\lim_{t \to \infty} \frac{S(t)}{1000l} = \lim_{t \to \infty} \frac{3kg + 4kge^{-\frac{1}{100min}t}}{1000l}$$

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$$S(10min) = 3kg + 4kge^{-\frac{1}{100min}10min}$$

= $3kg + 4kge^{-\frac{1}{10}} \approx 6.6193kg$
$$\lim_{t \to \infty} \frac{S(t)}{1000l} = \lim_{t \to \infty} \frac{3kg + 4kge^{-\frac{1}{100min}t}}{1000l}$$

= $\frac{3kg}{1000l}$

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Results appear sensible.

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min}$$

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg$$

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg \qquad \sqrt{2}$$

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg \qquad \sqrt{\frac{dS}{dt}} = \frac{d}{dt} \left(3kg + 4kge^{-\frac{1}{100min}t}\right)$$

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Results appear sensible.

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$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg \qquad \sqrt{\frac{dS}{dt}} = \frac{d}{dt} \left(3kg + 4kge^{-\frac{1}{100min}t}\right)$$
$$= 0 + 4kg \left(-\frac{1}{100min}\right)e^{-\frac{1}{100min}t}$$

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg \qquad \sqrt{\frac{dS}{dt}} = \frac{d}{dt} \left(3kg + 4kge^{-\frac{1}{100min}t}\right)$$
$$= 0 + 4kg \left(-\frac{1}{100min}\right)e^{-\frac{1}{100min}t}$$
$$= \frac{3kg - S(t)}{100min}$$

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Results appear sensible.

$$S(0) = 3kg + 4kge^{-\frac{1}{100min}0min} = 7kg \qquad \sqrt{\frac{dS}{dt}} = \frac{d}{dt} \left(3kg + 4kge^{-\frac{1}{100min}t}\right)$$
$$= 0 + 4kg \left(-\frac{1}{100min}\right)e^{-\frac{1}{100min}t}$$
$$= \frac{3kg - S(t)}{100min} \qquad \sqrt{$$

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Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{dS}{dt}$ =	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(0)$)) = 7kg

Overview	An Example	Double Check	Units Reprised
Solve the Initial Val	ue Problem $\frac{dS}{dt}$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(0)$)=7kg
	$\frac{dS}{dt} = \frac{3}{10}$	$\frac{3}{00}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min}$	

Overview	An Example			Doubl	e Check		Units Reprised
Solve the Initial Value	Problem	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{k}{m}$	<u>8</u> 1. 1.	$\frac{S(t)}{100} \frac{1}{min}$	$\frac{1}{2}, S(0) = 7k_{\xi}$	7
	$\frac{dS}{dt} =$	$\frac{3}{100}$	kg min	$\frac{S(t)}{100}$	$\frac{1}{min} =$	$\frac{3kg - S(t)}{100min}$	

Overview	An Example	Double Check	Units Reprised
Solve the Initial V	Value Problem $\frac{dS}{dt} =$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min}$	-n, S(0) = 7kg
$\overline{3k_{\delta}}$	$\frac{dS}{dt} = \frac{3}{100}$ $\frac{dS}{g-S(t)} = \frac{3}{100}$	$\frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min} = \frac{1}{min} dt$	$=\frac{3kg-S(t)}{100min}$

Overview	An Example	Double Check	Units Reprised
Solve the Initial Valu	e Problem $\frac{dS}{dt}$ =	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{s}$	$\frac{1}{\min}, S(0) = 7kg$
	dS = 3	kg = S(t) = 1	3kg - S(t)
	$\frac{dt}{dt} = \frac{100}{100}$	$\overline{min} = \overline{100} \overline{min}$	= 100min
dS	5	1	
$\overline{3kg}$ –	$\overline{S(t)} = \overline{100}$	min ^{at}	
$\int \frac{1}{3 - \frac{S(t)}{kg}}$	$d\frac{S}{kg} = \int \frac{1}{1}$	$\frac{1}{00min}$ dt	

Overview	An Example	Double Check	Units Reprised
Solve the Initial Valu	e Problem $\frac{dS}{dt}$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{m}$	$\frac{1}{nin}, S(0) = 7kg$
	dS = 3	kg = S(t) = 1	3kg - S(t)
	$\frac{dt}{dt} = \frac{10}{10}$	$\overline{0} \overline{min} - \overline{100} \overline{min}$	
dS	5	1	
$\overline{3kg}$ –	$\overline{S(t)} = \overline{10}$	Omin ^{dt}	
$\int \frac{1}{3 - \frac{S(t)}{kg}}$	$d\frac{S}{kg} = \int$	$\frac{1}{100min} dt$	
$-\ln \left 3 - \frac{1}{2} \right $	$\left \frac{S(t)}{kg}\right = \frac{10}{10}$	$\frac{1}{0min}t+c$	

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Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem	$n \frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0) = 7k$	g
	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min} = \frac{3kg - S(t)}{100min}$)
	ai dS	100 min 100 min 100 min	
-	$\frac{us}{1-u(s)} =$	$\frac{1}{100}$ dt	
37	kg - S(t)	100 <i>min</i>	
$\int \frac{1}{3-1}$	$\frac{1}{\frac{S(t)}{kg}} d\frac{S}{kg} =$	$\int \frac{1}{100min} dt$	
$-\ln \left \cdot \right $	$3 - \frac{S(t)}{kg} \bigg =$	$\frac{1}{100min}t+c$	
3	kg - S(t) =	$e^{-\frac{1}{100min}t-c} kg$	

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Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0)$)=7kg
	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min} = \frac{3kg}{100}$	$\frac{-S(t)}{0}$
	ai dS	100 min 100 min 100	omin
37	$\frac{dS}{kg - S(t)} =$	$\frac{1}{100min}$ dt	
$\int \frac{1}{3-1}$	$\frac{1}{\frac{S(t)}{kg}} d\frac{S}{kg} =$	$\int \frac{1}{100min} dt$	
$-\ln $	$3 - \frac{S(t)}{kg} \bigg =$	$\frac{1}{100min}t + c$	
3.	kg - S(t) =	$e^{-\frac{1}{100\min}t-c} kg = ke^{-\frac{1}{100\min}t} kg$	g

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Overview	An Example	Double Check	Units Reprised
Solve the Initial Value	Problem	$\frac{dS}{dt} = \frac{3}{100} \frac{kg}{min} - \frac{S(t)}{100} \frac{1}{min}, S(0) = 7k_0$	g
dS	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min} = \frac{3kg - S(t)}{100min}$	
$\overline{3kg-3}$	$\overline{S(t)} =$	$\frac{100min}{100min}$ dt	
$\int \frac{1}{3 - \frac{S(t)}{kg}} dt$	$d\frac{S}{kg} =$	$\int \frac{1}{100min} dt$	
$-\ln\left 3-\frac{S}{\mu}\right $	$\left \frac{(t)}{kg} \right =$	$\frac{1}{100min}t + c$	
3kg - b	S(t) =	$e^{-\frac{1}{100\min}t-c} kg = ke^{-\frac{1}{100\min}t} kg$	
	S(t) =	$3kg - ke^{-\frac{1}{100min}t} kg$	

Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{dS}{dt}$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(t)$	0) = 7kg

Overview	An Example		Dou	ıble Check	Units Reprised
Solve the Initia	l Value Problem	$\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min}$	$-\frac{S(t)}{100}\frac{1}{min}, S(0) = 1$	7kg
7 <i>k</i>	g = S(0)				

Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{dS}{dt} =$	$\frac{3}{100}\frac{kg}{min} - \frac{S(t)}{100}\frac{1}{min}, S(0)$) = 7kg
7 <i>k</i> {	g = S(0) = 3kg -	$ke^{-\frac{1}{100min}0}kg$	

Overview	An Example	Double Check	Units Reprised
Solve the Initia	1 Value Problem $\frac{dS}{dt}$ =	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(0)$)=7kg
7 <i>k</i>	g=S(0) = 3kg -	$-ke^{-\frac{1}{100min}0} kg = 3kg - k$	x kg

Overview	An Example	Double Check	Units Reprised
Solve the Initia	al Value Problem $\frac{dS}{dt} =$	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min}, S(0)$))=7kg
78	cg = S(0) = 3kg - k = -4	$-ke^{-\frac{1}{100min}0} kg = 3kg - kg$	k kg

Overview	An Example	Double Check	Units Reprised
Solve the Initial	Value Problem $\frac{dS}{dt}$ =	$=\frac{3}{100}\frac{kg}{min}-\frac{S(t)}{100}\frac{1}{min},S(0)$)=7kg
7kg	=S(0) = 3kg - kg - kg - 4kg - kg - 4kg -	$-ke^{-\frac{1}{100min}0}kg = 3kg - k$	t kg
	S(t) = 3kg +	$-4kge^{-\frac{1}{100min}t}$	