# Non-Diagonalizable Homogeneous Systems of Linear Differential Equations with Constant Coefficients

Bernd Schröder

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5. Conversely, every solution of $\vec{y}' = A\vec{y}$ can be obtained as above.

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9. The multiplicity of the eigenvalue $\lambda_j$ is the largest $k$ so that $(\lambda - \lambda_j)^k$ divides the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$.

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10. If the number of linearly independent eigenvectors for $\lambda_j$ is less than the multiplicity, then the matrix is not diagonalizable.

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11. If the multiplicity of $\lambda$ is at least 2, but the associated eigenspace is one dimensional, then $\vec{v}te^{\lambda t}+\vec{w}e^{\lambda t}$, with $\vec{v}$ being an eigenvector and $\vec{w}$ satisfying $(A-\lambda I)\vec{w}=\vec{v}$, is another, linearly independent, solution of $\vec{y}'=A\vec{y}$.

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12. If the multiplicity of $\lambda$ is at least 3, but the associated eigenspace is one dimensional, then $\vec{v}\frac{t^2}{2}e^{\lambda t}+\vec{w}te^{\lambda t}+\vec{x}e^{\lambda t}$, with $\vec{v}$ being an eigenvector, $\vec{w}$ satisfying $(A-\lambda I)\vec{w}=\vec{v}$, and $\vec{x}$ satisfying $(A-\lambda I)\vec{x}=\vec{w}$, is yet another linearly independent solution of $\vec{y}'=A\vec{y}$.

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13. There is more, but that's where matrix exponentials and the Jordan Normal Form make things more bearable.

# Solve the System $\vec{y}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \vec{y}$

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# Eigenvector for $\lambda = 2$

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