How to Classify First Order Equations and How to Remember the Solution Methods

Bernd Schröder

Some Natural Questions

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   2.1 Separable

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   - 2.2 Linear

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Recognizing Differential Equations

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Identify the type of the differential equation $x^2y' + xy - 1 = 0$.

$$\begin{aligned} x^2y' + xy - 1 &= 0 \\ x^2y' + xy &= 1 \\ y' + \frac{1}{x}y &= \frac{1}{x^2} \end{aligned}$$

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This is a linear equation.

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How would you solve this equation?

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$$\begin{aligned} y' &= e^x y - e^x + xy - x \\ y' &= e^x y + xy - e^x - x \end{aligned}$$

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Identify the type of the differential equation
$y' = e^x y - e^x + xy - x$.

$$\begin{aligned} y' &= e^x y - e^x + xy - x \\ y' &= e^x y + xy - e^x - x \\ y' &= (e^x + x)\, y \end{aligned}$$

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$$\begin{aligned} y' &= e^x y - e^x + xy - x \\ y' &= e^x y + xy - e^x - x \\ y' &= (e^x + x)\, y + (e^x + x)\,(-1) \end{aligned}$$

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This is a separable equation.

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This is a separable equation.
How would you solve this equation?

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This is a Bernoulli equation.

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Recognizing Differential Equations

Identify the type of the differential equation $y' = \dfrac{y^3 - y^2 x}{x^3}$.

$$
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y' &= \frac{y^3 - y^2 x}{x^3} \\
y' &= \frac{y^3}{x^3} - \frac{y^2}{x^2} \\
y' &= \left(\frac{y}{x}\right)^3 - \left(\frac{y}{x}\right)^2
\end{aligned}
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This is a homogeneous equation.

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This is a homogeneous equation.
How would you solve this equation?

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This is a homogeneous equation.
How would you solve this equation?
(This is the only equation in this presentation for which the solution does not work out nicely.)

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Recognizing Differential Equations

Identify the type of the differential equation $y' = \frac{y}{x} + 5$.

$$\begin{aligned} y' &= \frac{y}{x} + 5 \\ y' &= \frac{1}{x}y + 5 \\ y' - \frac{1}{x}y &= 5 \end{aligned}$$

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Identify the type of the differential equation $y' = \frac{y}{x} + 5$.

$$\begin{aligned} y' &= \frac{y}{x} + 5 \\ y' &= \frac{1}{x}y + 5 \\ y' - \frac{1}{x}y &= 5 \end{aligned}$$

This is a linear equation.

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How would you solve this equation?

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How would you solve this equation?

It is also a homogeneous equation: $y' = \left(\frac{y}{x}\right) + 5$.

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How would you solve this equation as a homogeneous equation?

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This is a linear equation.

How would you solve this equation?

It is also a homogeneous equation: $y' = \left(\frac{y}{x}\right) + 5$.

How would you solve this equation as a homogeneous equation?

Which way is easier?

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How would you solve this equation (either way)?

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