Translating Higher Order Equations/Systems to First Order Systems

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Intermediate Derivatives Become Dummy Variables

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Let $a_0(t), \ldots, a_{n-1}(t)$ and $g(t)$ be differentiable functions. The function y is a solution of the n^{th} order linear differential equation $y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots + a_1(t)y' + a_0(t)y = g(t)$ if and only if $y_0 := y$, $y_1 := y'$, $\ldots$, $y_{n-1} := y^{(n-1)}$ is a solution of the system of linear equations

$$\begin{aligned}
y_0' &= y_1 \\
y_1' &= y_2 \\
&\vdots \quad \vdots \\
y_{n-2}' &= y_{n-1} \\
y_{n-1}' &= -a_{n-1}(t)y_{n-1} - \cdots - a_1(t)y_1 - a_0(t)y_0 + g(t).
\end{aligned}$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

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$$y_1'$$

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Idea: $y_0 = y, y_1 = y'$

$$y_1 = y_0'$$

$$y_1' + 4y_1$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

Idea: $y_0 = y, y_1 = y'$

$$\begin{aligned} y_1 &= y_0' \\ y_1' + 4y_1 + 5y_0 \end{aligned}$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

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$$\begin{aligned} y_1 &= y_0' \\ y_1' + 4y_1 + 5y_0 &= 0 \end{aligned}$$

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$$\begin{aligned} y_1 &= y_0' \\ y_1' + 4y_1 + 5y_0 &= 0 \\ y_0(0) &= 0 \end{aligned}$$

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$$\begin{aligned} y_1 &= y_0' \\ y_1' + 4y_1 + 5y_0 &= 0 \\ y_0(0) &= 0 \\ y_1(0) &= 1 \end{aligned}$$

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$$\begin{aligned} y_0' &= y_1 \end{aligned}$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

$$\begin{aligned} y_0' &= y_1 \\ y_1' &= -4y_1 - 5y_0 \end{aligned}$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

$$\begin{aligned} y_0' &= y_1 \\ y_1' &= -4y_1 - 5y_0 \\ y_0(0) &= 0 \end{aligned}$$

Translate the Initial Value Problem $y'' + 4y' + 5y = 0, y(0) = 0, y'(0) = 1$ Into an IVP for a First Order System

$$\begin{aligned} y_0' &= y_1 \\ y_1' &= -4y_1 - 5y_0 \\ y_0(0) &= 0 \\ y_1(0) &= 1 \end{aligned}$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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Idea: $u_1 := y_1$, $u_2 := y_1'$, $u_3 := y_2$, $u_4 := y_2'$

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$$u_2 = u_1'$$

Translate the Initial Value Problem

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Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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$$u_2' + 3u_2 - u_1$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

Idea: $u_1 := y_1$, $u_2 := y_1'$, $u_3 := y_2$, $u_4 := y_2'$

$$u_2 = u_1'$$

$$u_2' + 3u_2 - u_1 + 2u_4$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

Idea: $u_1 := y_1$, $u_2 := y_1'$, $u_3 := y_2$, $u_4 := y_2'$

$$\begin{aligned} u_2 &= u_1' \\ u_2' + 3u_2 - u_1 + 2u_4 &= 0 \end{aligned}$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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$$\begin{aligned} u_2 &= u_1' \\ u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\ u_4 &= u_3' \end{aligned}$$

Translate the Initial Value Problem

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$$\begin{aligned} u_2 &= u_1' \\ u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\ u_4 &= u_3' \\ u_4' & \end{aligned}$$

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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 &
\end{aligned}
$$

Translate the Initial Value Problem

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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 &
\end{aligned}
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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 - 2u_1 &
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$$\begin{aligned} u_2 &= u_1' \\ u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\ u_4 &= u_3' \\ u_4' - 2u_3 + u_2 - 2u_1 &= 0 \end{aligned}$$

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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 - 2u_1 &= 0 \\
u_1(0) &= 0
\end{aligned}
$$

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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 - 2u_1 &= 0 \\
u_1(0) = 0 \qquad & u_2(0) = 1
\end{aligned}
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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 - 2u_1 &= 0 \\
u_1(0) = 0 \qquad & u_2(0) = 1 \\
u_3(0) = 2 &
\end{aligned}
$$

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\begin{aligned}
u_2 &= u_1' \\
u_2' + 3u_2 - u_1 + 2u_4 &= 0 \\
u_4 &= u_3' \\
u_4' - 2u_3 + u_2 - 2u_1 &= 0 \\
u_1(0) = 0 \qquad & u_2(0) = 1 \\
u_3(0) = 2 \qquad & u_4(0) = 3
\end{aligned}
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Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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$$u_1' = u_2$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= -3u_2 + u_1 - 2u_4 \end{aligned}$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= -3u_2 + u_1 - 2u_4 \\ u_3' &= u_4 \end{aligned}$$

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$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= -3u_2 + u_1 - 2u_4 \\ u_3' &= u_4 \\ u_4' &= 2u_3 - u_2 + 2u_1 \end{aligned}$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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\begin{aligned}
u_1' &= u_2 \\
u_2' &= -3u_2 + u_1 - 2u_4 \\
u_3' &= u_4 \\
u_4' &= 2u_3 - u_2 + 2u_1 \\
u_1(0) &= 0
\end{aligned}
$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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\begin{aligned}
u_1' &= u_2 \\
u_2' &= -3u_2 + u_1 - 2u_4 \\
u_3' &= u_4 \\
u_4' &= 2u_3 - u_2 + 2u_1 \\
u_1(0) = 0 \qquad & \qquad u_2(0) = 1
\end{aligned}
$$

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\begin{aligned}
u_1' &= u_2 \\
u_2' &= -3u_2 + u_1 - 2u_4 \\
u_3' &= u_4 \\
u_4' &= 2u_3 - u_2 + 2u_1
\end{aligned}
$$

$$
u_1(0) = 0 \qquad u_2(0) = 1
$$

$$
u_3(0) = 2
$$

Translate the Initial Value Problem

$y_1'' + 3y_1' - y_1 + 2y_2' = 0$, $y_2'' - 2y_2 + y_1' - 2y_1 = 0$, $y_1(0) = 0$, $y_1'(0) = 1$, $y_2(0) = 2$, $y_2'(0) = 3$ Into an IVP for a First Order System

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\begin{aligned}
u_1' &= u_2 \\
u_2' &= -3u_2 + u_1 - 2u_4 \\
u_3' &= u_4 \\
u_4' &= 2u_3 - u_2 + 2u_1
\end{aligned}
$$

$$
\begin{aligned}
u_1(0) &= 0 & u_2(0) &= 1 \\
u_3(0) &= 2 & u_4(0) &= 3
\end{aligned}
$$