The Method of Undetermined Coefficients

Bernd Schröder

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2. The Method of Undetermined Coefficients is a way to obtain a particular solution of the inhomogeneous equation.
3. The idea is to detect repeating patterns in the derivatives of the inhomogeneity and to set up the particular solution as a linear combination of the patterns with undetermined coefficients.

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4. Once the conjectured solution is set up, substitute it into the equation and determine the coefficients.

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2. The Method of Undetermined Coefficients is a way to obtain a particular solution of the inhomogeneous equation.
3. The idea is to detect repeating patterns in the derivatives of the inhomogeneity and to set up the particular solution as a linear combination of the patterns with undetermined coefficients.
4. Once the conjectured solution is set up, substitute it into the equation and determine the coefficients.

That's it.

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

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Solution of the homogeneous equation.

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$$y'' + 4y' + 4y = 0$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Solution of the homogeneous equation.

$$\begin{aligned} y''+4y'+4y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0 \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Solution of the homogeneous equation.

$$\begin{aligned} y''+4y'+4y &= 0 \\ \lambda^2 e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0 \\ \lambda^2+4\lambda+4 &= 0 \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Solution of the homogeneous equation.

$$\begin{aligned}
y''+4y'+4y &= 0\\
\lambda^2e^{\lambda x}+4\lambda e^{\lambda x}+4e^{\lambda x} &= 0\\
\lambda^2+4\lambda+4 &= 0\\
\lambda_{1,2} &= \frac{-4\pm\sqrt{16-16}}{2}
\end{aligned}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Solution of the homogeneous equation.

$$\begin{aligned} y'' + 4y' + 4y &= 0 \\ \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \\ \lambda^2 + 4\lambda + 4 &= 0 \\ \lambda_{1,2} &= \frac{-4 \pm \sqrt{16-16}}{2} = -2 \end{aligned}$$

Solve the Initial Value Problem
$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Solution of the homogeneous equation.

$$\begin{aligned}
y'' + 4y' + 4y &= 0 \\
\lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} &= 0 \\
\lambda^2 + 4\lambda + 4 &= 0 \\
\lambda_{1,2} &= \frac{-4 \pm \sqrt{16-16}}{2} = -2 \\
y_h &= c_1 e^{-2x} + c_2 x e^{-2x}
\end{aligned}$$

Solve the Initial Value Problem

$$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$$

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Generating the *form* of the particular solution of the inhomogeneous equation.

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Generating the *form* of the particular solution of the inhomogeneous equation.

$$r(x) = \sin(x)$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Generating the *form* of the particular solution of the inhomogeneous equation.

$$\begin{aligned} r(x) &= \sin(x) \\ &\quad \text{(need a sine term)} \end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

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$$\begin{aligned} r(x) &= \sin(x) \\ &\quad \text{(need a sine term)} \\ r'(x) &= \cos(x) \end{aligned}$$

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$$\begin{aligned} r(x) &= \sin(x) \\ &\quad \text{(need a sine term)} \\ r'(x) &= \cos(x) \\ &\quad \text{(need a cosine term)} \\ r''(x) &= -\sin(x) \end{aligned}$$

Solve the Initial Value Problem

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Generating the *form* of the particular solution of the inhomogeneous equation.

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Generating the *form* of the particular solution of the inhomogeneous equation.

$$\begin{aligned} r(x) &= \sin(x) \\ &\quad (\text{need a sine term}) \\ r'(x) &= \cos(x) \\ &\quad (\text{need a cosine term}) \\ r''(x) &= -\sin(x) \\ &\quad (\text{already have a sine term}) \\ &\quad (\text{after this, it ``repeats''}) \\ y_p &:= A\cos(x)+B\sin(x) \end{aligned}$$

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Solve the Initial Value Problem

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Determining the coefficients.

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

$$y''+4y'+4y = \sin(x)$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

$$y''+4y'+4\big(A\cos(x)+B\sin(x)\big) \;=\; \sin(x)$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Determining the coefficients.

$$y'' + 4\big(-A\sin(x) + B\cos(x)\big) + 4\big(A\cos(x) + B\sin(x)\big) \;=\; \sin(x)$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Determining the coefficients.

$$\Big(-A\cos(x)-B\sin(x)\Big)+4\Big(-A\sin(x)+B\cos(x)\Big)+4\Big(A\cos(x)+B\sin(x)\Big) \;=\; \sin(x)$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

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$$(-A+4B+4A)\cos(x)$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

$$\begin{aligned}\big(-A\cos(x)-B\sin(x)\big)+4\big(-A\sin(x)+B\cos(x)\big)+4\big(A\cos(x)+B\sin(x)\big) &= \sin(x)\\ (-A+4B+4A)\cos(x)+(-B-4A+4B)\sin(x)\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Determining the coefficients.

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\big(-A\cos(x)-B\sin(x)\big)+4\big(-A\sin(x)+B\cos(x)\big)+4\big(A\cos(x)+B\sin(x)\big) &= \sin(x)\\
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Solve the Initial Value Problem
$y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

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\big(-A\cos(x)-B\sin(x)\big)+4\big(-A\sin(x)+B\cos(x)\big)+4\big(A\cos(x)+B\sin(x)\big) &= \sin(x)\\
(-A+4B+4A)\cos(x)+(-B-4A+4B)\sin(x) &= \sin(x)\\
(3A+4B)\cos(x)+(-4A+3B)\sin(x) &= \sin(x)
\end{aligned}$$

Solve the Initial Value Problem
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(3A+4B)\cos(x)+(-4A+3B)\sin(x) &= \sin(x)\\
3A+4B=0\\
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(3A+4B)\cos(x)+(-4A+3B)\sin(x) &= \sin(x)
\end{aligned}$$

$$3A+4B=0$$

$$-4A+3B=1$$

$$12A+16B=0$$

Solve the Initial Value Problem
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Determining the coefficients.

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\big(-A\cos(x)-B\sin(x)\big)+4\big(-A\sin(x)+B\cos(x)\big)+4\big(A\cos(x)+B\sin(x)\big) &= \sin(x)\\
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\end{aligned}$$

$$3A+4B=0$$

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$$-12A+9B=3$$

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Determining the coefficients.

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\end{aligned}$$

$$3A+4B=0$$

$$-4A+3B=1$$

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$$-12A+9B=3$$

$$25B=3$$

Solve the Initial Value Problem
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(3A+4B)\cos(x)+(-4A+3B)\sin(x) &= \sin(x)
\end{aligned}$$

$$3A+4B=0$$

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$$25B=3$$

$$B=\frac{3}{25}$$

Solve the Initial Value Problem $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Determining the coefficients.

$$\begin{aligned}
\big(-A\cos(x) - B\sin(x)\big) + 4\big(-A\sin(x) + B\cos(x)\big) + 4\big(A\cos(x) + B\sin(x)\big) &= \sin(x) \\
(-A + 4B + 4A)\cos(x) + (-B - 4A + 4B)\sin(x) &= \sin(x) \\
(3A + 4B)\cos(x) + (-4A + 3B)\sin(x) &= \sin(x)
\end{aligned}$$

$$3A + 4B = 0$$

$$-4A + 3B = 1$$

$$12A + 16B = 0$$

$$-12A + 9B = 3$$

$$25B = 3$$

$$B = \frac{3}{25}, \quad A = -\frac{4}{3}B$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Determining the coefficients.

$$\begin{aligned}
\big(-A\cos(x)-B\sin(x)\big)+4\big(-A\sin(x)+B\cos(x)\big)+4\big(A\cos(x)+B\sin(x)\big) &= \sin(x)\\
(-A+4B+4A)\cos(x)+(-B-4A+4B)\sin(x) &= \sin(x)\\
(3A+4B)\cos(x)+(-4A+3B)\sin(x) &= \sin(x)
\end{aligned}$$

$$3A+4B=0$$

$$-4A+3B=1$$

$$12A+16B=0$$

$$-12A+9B=3$$

$$25B=3$$

$$B=\frac{3}{25},\quad A=-\frac{4}{3}B=-\frac{4}{25}$$

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Stating the General Solution.

Solve the Initial Value Problem
$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Stating the General Solution.

$$y_p = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x)$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Stating the General Solution.

$$\begin{aligned} y_p &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) \\ y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \end{aligned}$$

Solve the Initial Value Problem

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$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$

Finding c_1, c_2.

$$y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned} y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\ y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right) \end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0)
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

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\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
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1 &= y(0) = -\frac{4}{25} + c_1
\end{aligned}
$$

Solve the Initial Value Problem
$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25}
\end{aligned}$$

Solve the Initial Value Problem

$y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$

Finding c_1, c_2.

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y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\
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1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25} \\
0 &= y'(0)
\end{aligned}$$

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Finding c_1, c_2.

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1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25} \\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2
\end{aligned}$$

Solve the Initial Value Problem
$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

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y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right)\\
1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25}\\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2, \qquad c_2=2c_1-\frac{3}{25}
\end{aligned}$$

Solve the Initial Value Problem $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\
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1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25} \\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2, \qquad c_2=2c_1-\frac{3}{25}=\frac{55}{25}
\end{aligned}$$

Solve the Initial Value Problem

$y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$

Finding c_1, c_2.

$$\begin{aligned}
y &= -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + c_1 e^{-2x} + c_2 x e^{-2x} \\
y' &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - 2c_1 e^{-2x} + c_2\left(e^{-2x} - 2xe^{-2x}\right) \\
1 &= y(0) = -\frac{4}{25} + c_1, \qquad c_1 = \frac{29}{25} \\
0 &= y'(0) = \frac{3}{25} - 2c_1 + c_2, \qquad c_2 = 2c_1 - \frac{3}{25} = \frac{55}{25} = \frac{11}{5}
\end{aligned}$$

Solve the Initial Value Problem

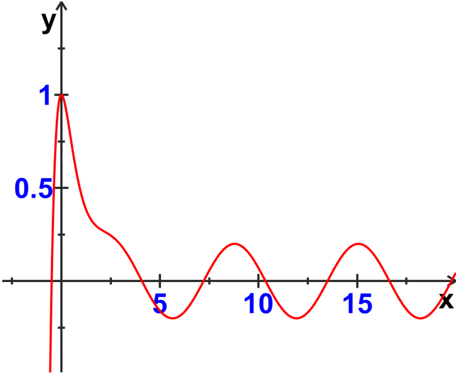
$y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$

Finding c_1, c_2.

$$
\begin{aligned}
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+c_1e^{-2x}+c_2xe^{-2x} \\
y' &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-2c_1e^{-2x}+c_2\left(e^{-2x}-2xe^{-2x}\right) \\
1 &= y(0)=-\frac{4}{25}+c_1, \qquad c_1=\frac{29}{25} \\
0 &= y'(0)=\frac{3}{25}-2c_1+c_2, \qquad c_2=2c_1-\frac{3}{25}=\frac{55}{25}=\frac{11}{5} \\
y &= -\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}
\end{aligned}
$$

Solve the Initial Value Problem

$$y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$$



$$y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
& 4\left(-\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}\right) \\
+4 & \left(\frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{11}{5}\left(e^{-2x} - 2xe^{-2x}\right)\right)
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$
\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x),\ y(0)=1,\ y'(0)=0$?

$$
\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \overset{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$
\begin{aligned}
& 4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right) \\
& +4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right) \\
& +\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \quad \stackrel{?}{=} \quad \sin(x) \\
& \left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x) \\
& +\left(\frac{116}{25}-\frac{232}{25}+\frac{44}{5}+\frac{116}{25}-\frac{44}{5}\right)e^{-2x}
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned}
&4\left(-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}\right)\\
&+4\left(\frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{11}{5}\left(e^{-2x}-2xe^{-2x}\right)\right)\\
&+\frac{4}{25}\cos(x)-\frac{3}{25}\sin(x)+\frac{116}{25}e^{-2x}+\frac{11}{5}\left(-4e^{-2x}+4xe^{-2x}\right) \stackrel{?}{=} \sin(x)\\
&\left(-\frac{16}{25}+\frac{12}{25}+\frac{4}{25}\right)\cos(x)+\left(\frac{12}{25}+\frac{16}{25}-\frac{3}{25}\right)\sin(x)\\
&+\left(\frac{116}{25}-\frac{232}{25}+\frac{44}{5}+\frac{116}{25}-\frac{44}{5}\right)e^{-2x}+\left(\frac{44}{5}-\frac{88}{5}+\frac{44}{5}\right)xe^{-2x}
\end{aligned}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x),\ y(0) = 1,\ y'(0) = 0$?

$$4\left(-\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}\right)$$
$$+4\left(\frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{11}{5}\left(e^{-2x} - 2xe^{-2x}\right)\right)$$
$$+\frac{4}{25}\cos(x) - \frac{3}{25}\sin(x) + \frac{116}{25}e^{-2x} + \frac{11}{5}\left(-4e^{-2x} + 4xe^{-2x}\right) \stackrel{?}{=} \sin(x)$$
$$\left(-\frac{16}{25} + \frac{12}{25} + \frac{4}{25}\right)\cos(x) + \left(\frac{12}{25} + \frac{16}{25} - \frac{3}{25}\right)\sin(x)$$
$$+\left(\frac{116}{25} - \frac{232}{25} + \frac{44}{5} + \frac{116}{25} - \frac{44}{5}\right)e^{-2x} + \left(\frac{44}{5} - \frac{88}{5} + \frac{44}{5}\right)xe^{-2x} \stackrel{?}{=} \sin(x)$$

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\begin{aligned}
& 4\left(-\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}\right) \\
& +4\left(\frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{11}{5}\left(e^{-2x} - 2xe^{-2x}\right)\right) \\
& +\frac{4}{25}\cos(x) - \frac{3}{25}\sin(x) + \frac{116}{25}e^{-2x} + \frac{11}{5}\left(-4e^{-2x} + 4xe^{-2x}\right) \stackrel{?}{=} \sin(x) \\
& \left(-\frac{16}{25} + \frac{12}{25} + \frac{4}{25}\right)\cos(x) + \left(\frac{12}{25} + \frac{16}{25} - \frac{3}{25}\right)\sin(x) \\
& +\left(\frac{116}{25} - \frac{232}{25} + \frac{44}{5} + \frac{116}{25} - \frac{44}{5}\right)e^{-2x} + \left(\frac{44}{5} - \frac{88}{5} + \frac{44}{5}\right)xe^{-2x} \stackrel{\surd}{=} \sin(x)
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

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$$y(0) = -\frac{4}{25}+\frac{29}{25}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$y(0) = -\frac{4}{25}+\frac{29}{25}=1$$

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$$y(0) \quad = \quad -\frac{4}{25}+\frac{29}{25}=1 \quad \surd$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

$$\begin{aligned} y(0) &= -\frac{4}{25}+\frac{29}{25}=1 \quad \surd \\ y'(x) &= \frac{4}{25}\sin(x)+\frac{3}{25}\cos(x)-\frac{58}{25}e^{-2x}+\frac{55}{25}\left(e^{-2x}-2xe^{-2x}\right) \end{aligned}$$

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$
\begin{aligned}
y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\
y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\
y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25}
\end{aligned}
$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

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y'(0) &= \frac{3}{25}-\frac{58}{25}+\frac{55}{25}=0
\end{aligned}
$$

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y'(0) &= \frac{3}{25}-\frac{58}{25}+\frac{55}{25}=0 \quad \surd
\end{aligned}$$

Does $y=-\frac{4}{25}\cos(x)+\frac{3}{25}\sin(x)+\frac{29}{25}e^{-2x}+\frac{11}{5}xe^{-2x}$ Solve $y''+4y'+4y=\sin(x)$, $y(0)=1$, $y'(0)=0$?

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y(0) &= -\frac{4}{25}+\frac{29}{25}=1 \quad \surd \\
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y'(0) &= \frac{3}{25}-\frac{58}{25}+\frac{55}{25}=0 \quad \surd
\end{aligned}$$

Alternatively, use a computer to double check the result.

Does $y = -\frac{4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}xe^{-2x}$ Solve $y'' + 4y' + 4y = \sin(x)$, $y(0) = 1$, $y'(0) = 0$?

$$\begin{aligned} y(0) &= -\frac{4}{25} + \frac{29}{25} = 1 \quad \surd \\ y'(x) &= \frac{4}{25}\sin(x) + \frac{3}{25}\cos(x) - \frac{58}{25}e^{-2x} + \frac{55}{25}\left(e^{-2x} - 2xe^{-2x}\right) \\ y'(0) &= \frac{3}{25} - \frac{58}{25} + \frac{55}{25} = 0 \quad \surd \end{aligned}$$

Alternatively, use a computer to double check the result. Beyond a certain level of complexity, that really is the way to go.

Double Checking with a Computer Algebra System

Double Checking with a Computer Algebra System

$$y(x) := \frac{-4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}x \cdot e^{-2x}$$

Double Checking with a Computer Algebra System

$$y(x) := \frac{-4}{25}\cos(x) + \frac{3}{25}\sin(x) + \frac{29}{25}e^{-2x} + \frac{11}{5}x\cdot e^{-2x}$$

$$\frac{d^2}{dx^2}y(x) + 4\cdot\frac{d}{dx}y(x) + 4\cdot y(x) \text{ simplify } \rightarrow \sin(x)$$

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$$\frac{d^2}{dx^2}y(x) + 4\cdot\frac{d}{dx}y(x) + 4\cdot y(x) \text{ simplify } \rightarrow \sin(x)$$

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$$\frac{d^2}{dx^2}y(x) + 4\cdot\frac{d}{dx}y(x) + 4\cdot y(x) \text{ simplify } \rightarrow \sin(x)$$

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$$yp(x) := \frac{d}{dx}y(x)$$

Double Checking with a Computer Algebra System

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$$\frac{d^2}{dx^2}y(x) + 4\cdot\frac{d}{dx}y(x) + 4\cdot y(x) \text{ simplify } \rightarrow \sin(x)$$

$$y(0) \text{ simplify } \rightarrow 1$$

$$yp(x) := \frac{d}{dx}y(x)$$

$$yp(0) \text{ simplify } \rightarrow 0$$

How to Generate the Setup for Certain Right Sides

How to Generate the Setup for Certain Right Sides

1. Compute derivatives of the right side until no new patters emerge.

How to Generate the Setup for Certain Right Sides

1. Compute derivatives of the right side until no new patters emerge.
2. Generate a term (with an undetermined coefficient) for every term that occurs in one of the derivatives.

How to Generate the Setup for Certain Right Sides

How to Generate the Setup for Certain Right Sides

$$r(x) = x^2$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2 \\ r'(x) &= 2x \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2 \\ r'(x) &= 2x \\ r''(x) &= 2 \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2 \\ r'(x) &= 2x \\ r''(x) &= 2 \quad \text{(zeros from here)} \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$
\begin{aligned}
r(x) &= x^2 \\
r'(x) &= 2x \\
r''(x) &= 2 \quad \text{(zeros from here)}
\end{aligned}
$$

$$
y_p = Ax^2
$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2 \\ r'(x) &= 2x \\ r''(x) &= 2 \quad \text{(zeros from here)} \end{aligned}$$

$$y_p = Ax^2 + Bx$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2 \\ r'(x) &= 2x \\ r''(x) &= 2 \quad \text{(zeros from here)} \end{aligned}$$

$$y_p = Ax^2 + Bx + C$$

How to Generate the Setup for Certain Right Sides

How to Generate the Setup for Certain Right Sides

$$r(x) = x^2 e^x$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2e^x \\ r'(x) &= 2xe^x + x^2e^x \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2e^x \\ r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$
\begin{aligned}
r(x) &= x^2e^x \\
r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\
r(x) &= 2e^x + 4xe^x + x^2e^x
\end{aligned}
$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2e^x \\ r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\ r(x) &= 2e^x + 4xe^x + x^2e^x \quad \text{(only the first term is new)} \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$\begin{aligned} r(x) &= x^2e^x \\ r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\ r(x) &= 2e^x + 4xe^x + x^2e^x \quad \text{(only the first term is new)} \\ & \quad \text{(further derivatives do not generate new terms)} \end{aligned}$$

How to Generate the Setup for Certain Right Sides

$$
\begin{aligned}
r(x) &= x^2e^x \\
r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\
r(x) &= 2e^x + 4xe^x + x^2e^x \quad \text{(only the first term is new)} \\
&\quad \text{(further derivatives do not generate new terms)}
\end{aligned}
$$

$$
y_p = Ax^2e^x
$$

How to Generate the Setup for Certain Right Sides

$$
\begin{aligned}
r(x) &= x^2e^x \\
r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\
r(x) &= 2e^x + 4xe^x + x^2e^x \quad \text{(only the first term is new)} \\
& \text{(further derivatives do not generate new terms)}
\end{aligned}
$$

$$
y_p = Ax^2e^x + Bxe^x
$$

How to Generate the Setup for Certain Right Sides

$$
\begin{aligned}
r(x) &= x^2e^x \\
r'(x) &= 2xe^x + x^2e^x \quad \text{(only the first term is new)} \\
r(x) &= 2e^x + 4xe^x + x^2e^x \quad \text{(only the first term is new)} \\
&\quad \text{(further derivatives do not generate new terms)}
\end{aligned}
$$

$$
y_p = Ax^2e^x + Bxe^x + Ce^x
$$