Counting Techniques

Bernd Schröder
Equally Likely Outcomes

Permutations and Combinations

Examples

Introduction

1. In certain situations, all outcomes are equally likely:
   - Flipping a coin
   - Rolling dice
   - Dealing cards
   - Pulling different colored balls from an urn (the last one is a standard thought experiment in probability).

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**Theorem.** Let $\mathcal{S}$ be a sample space with a probability function $P$ so that every individual outcome/element in $\mathcal{S}$ has the same probability.

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**Theorem.** Let $\mathcal{I}$ be a sample space with a probability function $P$ so that every individual outcome/element in $\mathcal{I}$ has the same probability. Then the probability of an event $A$ is equal to the number of elements in $A$ divided by the number of elements in $\mathcal{I}$.

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Example. When 5 cards are dealt in a poker hand, the deal can be modeled as an ordered 5-tuple of cards: (first card [52 possibilities], second card [51 possibilities], third card [50 possibilities], fourth card [49 possibilities], fifth card [48 possibilities]), for a total of $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = 311,875,200$ possible ways the deal could happen.
**Theorem.** If there are $n_1$ ways to choose the first object, $n_2$ ways to choose the second, etc. and $n_k$ ways to choose the $k^{th}$ object, then there are $n_1 \cdot n_2 \cdots n_k$ ordered $k$-tuples.

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**Definition.**

An ordered sequence of \( k \) objects out of \( n \) distinct objects is called a permutation of size \( k \) of \( n \) objects.

The number of permutations of size \( k \) of \( n \) objects is denoted \( P_{k,n} \).

**Definition.**

For any nonnegative integer \( m \), we define the factorial to be:

\[
\begin{align*}
0! &= 1, \\
m! &= m \cdot (m-1) \cdot (m-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1, \\
&\quad \text{for } m > 0.
\end{align*}
\]

Theorem.

\[
P_{k,n} = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1) = \frac{n!}{(n-k)!},
\]

where \( n \) ≥ \( k \).

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We also define \( 0! = 1 \). (This makes certain formulas consistently applicable for "borderline cases".)

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1. Each suit has 13 cards.
2. If all cards come from the same suit, then we have \( \binom{13}{5} \) ways to get all 5 cards from that suit.
3. There are 4 suits.
4. So the number of possible flushes is \( 4 \cdot \binom{13}{5} = 5148 \).
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Permutations and Combinations
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Let's assume that aces can be high or low.

1. There are 10 ways to get a straight: Ace through 5 to 10 through ace.
2. Because there are 4 suits, there are 4 possibilities for each card in a straight that runs from one value to another.
3. Because of the way we count here, we don't need to divide out permutations.
4. So the number of possible straights is $10 \cdot 4 = 10\cdot 240$.

And that's why a flush beats a straight.
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Wild Bill Hickock and the Dead Man’s Hand
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Always remember that I endorse the *understanding* of games of chance
Wild Bill Hickock and the Dead Man’s Hand

Always remember that I endorse the *understanding* of games of chance, not gambling.