Predator-Prey Interaction Model with Hunting Cooperation Among Predators and Allee Effect in Prey

Aaditya Kharel; Michelle McCullum; Nick Burks
School of Mathematics and Natural Sciences, The University of Southern Mississippi

Abstract
Social interactions between individuals are an integral part of life history traits for many species (Courchamp et al., 2008). It is important that the predators and prey in any ecosystem coexist such that the presence of either of the species is not detrimental to their coexistence. Rather, the presence of both predator and prey species should provide a check and balance mechanism for each species to coexist and maintain their population in the ecosystem towards stable equilibrium. The Allee effect plays a significant role in determining the population dynamics of predator and prey to create stable coexistence equilibrium. We proposed a dynamical system of predator-prey interaction model consisting of two ordinary differential equations (ODEs) that incorporated hunting cooperation among the predators into a model of Rao and Kang. We investigate if hunting cooperation causes any Allee effect phenomena in prey. This is particularly important because if Allee effect is present in a two-species interaction model, we can predict that prey population below certain threshold will not survive at all in any ecosystem due to Allee effect.

Model
$$\begin{align*}
\frac{du}{dt} &= a(1-v)u - cu(1+v)u \\
\frac{dv}{dt} &= v(1+u) - du
\end{align*}$$

$u$ is the Prey Population
$v$ is the Predator Population
$a$ is the intrinsic growth rate of the Predator
$b$ is the Allee threshold $\in (-1, 1)$
$c$ is the energy conversion coefficient
d is the death rate of the Prey
$a$ is the hunting cooperation

Equilibrium Solutions

$$(0, 0), (1, 0), (0, 0), (u_1, v_1), (u_2, v_2)$$

$(0, 0), (1, 0)$ and $(0, 0)$ are our equilibrium boundary solutions.
$(u_1, v_1), (u_2, v_2)$ are our equilibrium interior solutions whose value is dependent on the choice of parameters.

Stability Analysis for Equilibrium Coexistence

$$\begin{align*}
\frac{dg_1}{du} &= -a(2u-b+1) + \frac{v(1+v)}{u(1+v)} < 0 \\
\frac{dg_2}{du} &= \frac{a(1+v)}{u(1+v)} > 0 \\
\frac{dg_3}{dv} &= -c(1+v) < 0
\end{align*}$$

Theorem 1: The interior solution is stable if $\frac{dg_3}{dv} < 0$ for $u, b \in \mathbb{R}^+$

Proof:
The interior solution is stable if and only if $\text{trace}(J) < 0$ and $\text{det}(J) > 0$ Suppose that $\frac{dg_3}{dv} < 0$ and $u, b \in \mathbb{R}^+$. Then, $\text{trace}(J) = f_{11} + f_{22}$ is negative. Similarly, $\text{det}(J) = f_{11}f_{22} - f_{12}f_{21}$ is positive because $f_{11}, f_{22} > 0$ and $-f_{12}f_{21} > 0$. Hence, it suffices to have $\frac{dg_3}{dv} < 0$ for stability of interior points.

$$f_1 = \left(1-a-b(1-u) - \frac{(1+a)v}{1+a} + \frac{1}{1+a}v\right)u = 0$$

Since $u + b, 1 > 0$

$$f_2 = \left(1-a-b(1-u) - \frac{(1+a)v}{1+a} + \frac{1}{1+a}v\right)v = 0$$

Since $v + b, 1 < 0$

Numerical Experiments

**Intersections**

![Figure 1](image1.png)

![Figure 2](image2.png)

**Predator-Prey vs Alpha**

![Figure 3](image3.png)

![Figure 4](image4.png)

Discussion

It was seen that when hunting cooperation, $a$, is 0.96, there is a change in stability of the equilibrium coexistence suggesting a bifurcation behavior. Any point above this hunting cooperation value is unstable and leads towards the extinction of the prey while any point below this value leads to the coexistence of both predators and prey. In our numerical simulation for the choices of parameter, another form of bifurcation behavior was noticed where there was a change in the number equilibrium coexistence as Allee Threshold ($b$) was increased.

Conclusions

We present a proof for the sufficient condition to ensure stability of the interior solutions, also known as the equilibrium coexistence points.

In order to study if hunting cooperation induces any Allee effect phenomena, we plan to conduct two-parameter solution analysis of hunting cooperation versus threshold for our choices of parameter.

With this two-parameter analysis, it might be suggestive if Allee effect is induced by hunting cooperation.

Future Directions

This graph suggests that there is a clear Allee threshold that indicates that prey population below certain threshold (between 0.2 and 0.4) population will cause both species to extinct and the prey population above the threshold will approach the equilibrium coexistence. Further research is necessary to see if hunting cooperation induces Allee effect phenomena.

References
